Nonlinear Control Theory
Introduction

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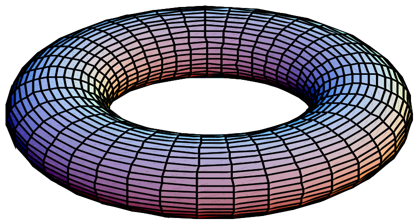

Outline

- Course resources
- Overview
- What is nonlinear control?
  - Linearization
- Why nonlinear control?
- Some examples

Resources

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URL: http://www.pages.drexel.edu/faculty/hgk22

Text Books & Software:

Kwatny, H. G. and Blankenship, "Nonlinear Control & Analytical Mechanics," Birkhauser, 2000 – Obtain update from H. Kwatny

Mathematica, Student Version 5.0

Requirements:

Problem sets

Final: Take-home project

Other References.

1. Slotine, J-J. E. and Li, W., "Applied Nonlinear Control," Prentice-Hall, 1991.
2. Vidyasagar, M., "Nonlinear Systems Analysis 2nd edition," Prentice-Hall, 1993.
3. Isidori, Alberto, "Nonlinear Control Systems-3rd edition," Springer-Verlag, 1995.
4. Nijmeijer, H. and H. J. van der Schaft, 1990: *Nonlinear Dynamical Control Systems*. Springer–Verlag.
5. Khalil, H. K., 1996: *Nonlinear Systems-2nd edition*. MacMillan.

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MEM 636 ~ Part I

- Introduction
  - *Course Overview, Using Mathematica*
- Nonlinear Dynamics, Stability
  - *The State Space, Equilibria & Stability, Hartman-Grobman Theorem*
  - *Stability ~ Liapunov Methods*
- Geometric Foundations
  - *Manifolds, Vector Fields & Integral Curves*
  - *Distributions, Frobenius Theorem & Integral Surfaces*
  - *Coordinate Transformations*
- Controllability & Observability
  - *Controllability & Observability, Canonical Forms*
- Stabilization via Feedback Linearization
  - *Linearization via Feedback*
  - *Stabilization using IO Linearization, Gain Scheduling*
- *Robust & Adaptive Control*
- *Tracking & Disturbance Rejection*

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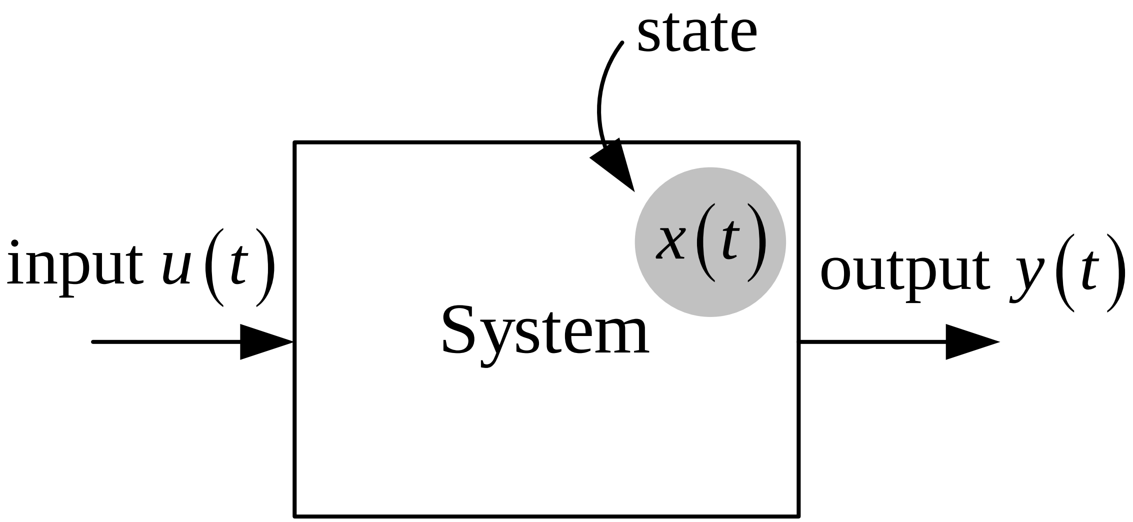
General Model of Nonlinear System



$$\dot{x} = f(x,u) \quad \text{state equation}$$

$$y = h(x,u) \quad \text{output equation}$$

$$x \in R^n, u \in R^m, y \in R^q$$

Special Case: Linear System

Nonlinear System

$$\dot{x} = f\left(x, u\right)$$

$$y = h\left(x, u\right)$$

Linear System

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

- Most real systems are nonlinear
- Sometimes a linear approximation is satisfactory
- Linear systems are much easier to analyze

Linear Systems are Nice

1. **Superposition Principle**: A linear combination of any two solutions for a linear system is also a solution.
2. **Unique Equilibrium**: Recall that an *equilibrium* is a solution $x(t)$, with $u(t) = 0$, for which x is constant. A generic Linear system has a unique (isolated) equilibrium at the origin $x(t) = 0$ and its stability is easily determined.
3. **Controllability**: There are known necessary and sufficient conditions under which a control exists to steer the state of a linear system from any initial value to any desired final value in finite time.
4. **Observability**: There are known necessary and sufficient conditions under which the system's state history can be determined from its input and output history.
5. **Control design tools**: A variety of controller and observer design techniques exist for linear systems (including classical techniques, pole placement, LQR/LQG, H_{inf}, etc.)

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Why Nonlinear Control

- Contemporary control problems require it,
  - Robotics, ground vehicles, propulsion systems, electric power systems, aircraft & spacecraft, autonomous vehicles, manufacturing processes, chemical & material processing,…
- Smooth (soft) nonlinearities
  - the system motion may not remain sufficiently close to an equilibrium point that the linear approximation is valid.
  - Also, linearization often removes essential physical effects – like Coriolis forces.
  - The optimal control may make effective use of nonlinearities
  - Robotics, process systems, spacecraft
- Non-smooth (hard) nonlinearities
  - Saturation, backlash, deadzone, hysteresis, friction, switching
- Systems that are not linearly controllable/observable may be controllable/observable in a nonlinear sense
  - Nonholonomic vehicles (try parking a car with a linear controller), underactuated mechanical systems (have fewer controls than dof), compressors near stall, ground vehicles near directional stability limit
- Systems that operate near instability (bifurcation points)
  - Power system voltage collapse, aircraft stall & spin, compressor surge & rotating stall, auto directional, cornering & roll stability
- Parameter adaptive & intelligent systems are inherently nonlinear

Linearization 1: linear approximation near an equilibrium point

The general nonlinear system in standard form, involves the state vector x, input vector u, and output vector y.

$$\dot{x} = f(x,u) \text{ state equations}$$

$$y = h(x,u) \text{ output equations}$$

A set of values (x_0, u_0, y_0) is called an **equilibrium point** if they satisfy:

$$0 = f(x_0, u_0)$$

$$y_0 = h(x_0, u_0)$$

We are interested in motions that remain close to the equilibrium point.

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Linearization 2

Define: $x(t) = x_0 + \delta x(t), u(t) = u_0 + \delta u(t), y(t) = y_0 + \delta y(t)$

The equations become:
$$\delta \dot{x} = f\left(x_0 + \delta x(t), u_0 + \delta u(t)\right)$$
$$y_0 + \delta y(t) = h\left(x_0 + \delta x(t), u_0 + \delta u(t)\right)$$

Now, construct a Taylor series for f, h

$$f\left(x_0 + \delta x, u_0 + \delta u\right) = f\left(x_0, u_0\right) + \frac{\partial f\left(x_0, u_0\right)}{\partial x}\delta x + \frac{\partial f\left(x_0, u_0\right)}{\partial u}\delta u + hot$$

$$h\left(x_0 + \delta x, u_0 + \delta u\right) = h\left(x_0, u_0\right) + \frac{\partial h\left(x_0, u_0\right)}{\partial x}\delta x + \frac{\partial h\left(x_0, u_0\right)}{\partial u}\delta u + hot$$

Notice that $f\left(x_0, u_0\right) = 0$ and $h\left(x_0, u_0\right) = y_0$, so

$$\begin{aligned} \delta \dot{x} &= \frac{\partial f\left(x_0, u_0\right)}{\partial x}\delta x + \frac{\partial f\left(x_0, u_0\right)}{\partial u}\delta u \\ \delta y &= \frac{\partial h\left(x_0, u_0\right)}{\partial x}\delta x + \frac{\partial h\left(x_0, u_0\right)}{\partial u}\delta u \end{aligned} \quad \Rightarrow \quad \boxed{\begin{aligned} \delta \dot{x} &= A\delta x + B\delta u \\ \delta y &= C\delta x + D\delta u \end{aligned}}$$

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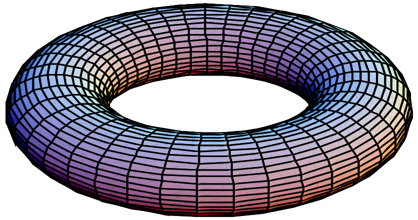
Some Examples of Nonlinear Systems



Example 1: Fully-Actuated Robotic System

q coordinates

p quasi-velocities

Models always look like:

kinematics: $\dot{q} = V(q)\,p$

dynamics: $M(q)\,\dot{p} + C(q,p)\,p + F(q,p) = u$

$$\det M(q) \neq 0 \quad \forall q$$

If we want to regulate velocity, choose

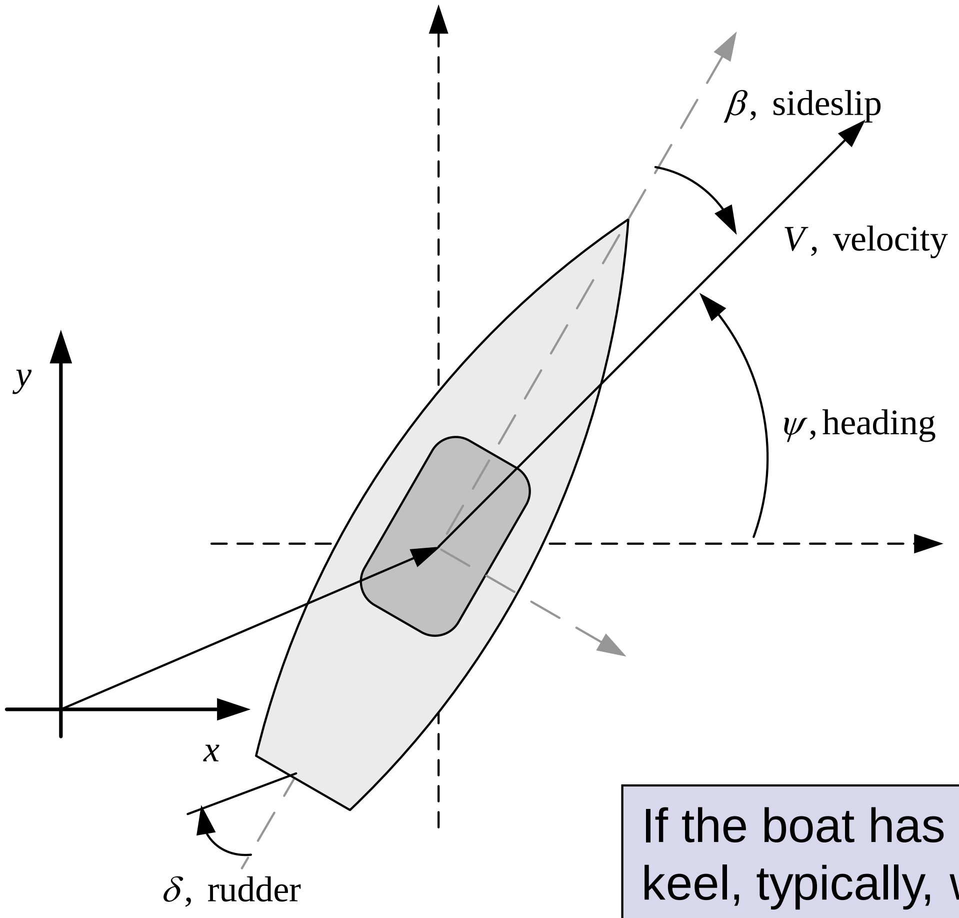
$$u = \left[C(q,p)\,p + F(q,p)\right] + M(q)\,v \Rightarrow \boxed{\dot{p} = v,\ \text{a linear system!}}$$

Decoupling

Stabilizing

If we want to regulate position, we can - more algebra. This is called the computed 'torque method.' We will generalize it.

Example 2



If the boat has a keel, typically, we have sideslip v=0.

Velocities in inertial coordinates

Velocities in body coordinates

kinematics: $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix}$

Consider $u_1 = u$ and $u_2 = r$ to be control inputs and suppose $v = 0$.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi \\ \sin\psi \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

Linearize about the origin $(x, y, \Psi) = (0,0,0)$ to get

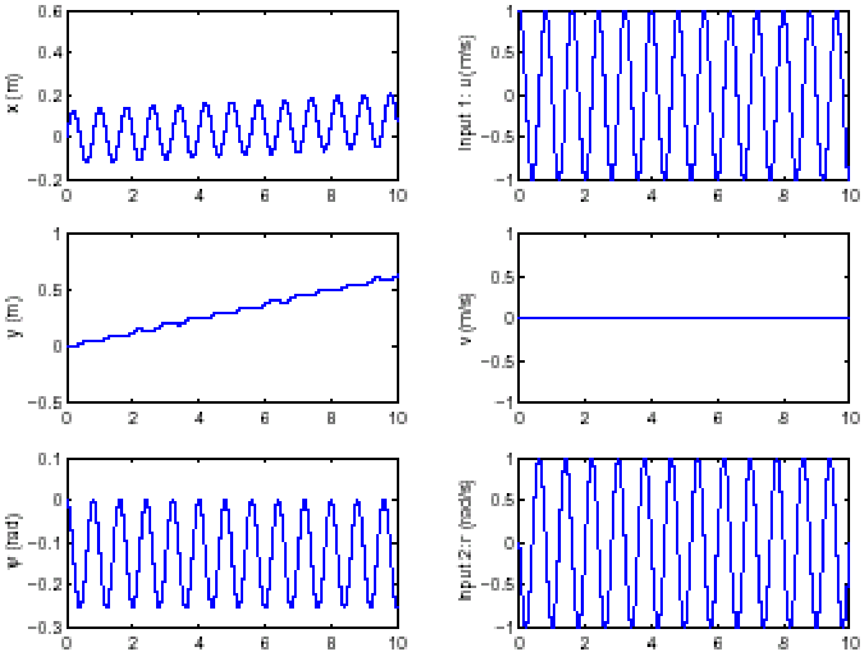
$\dot{x} = Ax + Bu, \quad A = 0,\ B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$ The linear system is not controllable! In particular, $\dot{y} = 0$, so it is not possible to steer from the origin along the y-axis

Example 2, cont'd

But, of course, if you allow large motions you can steer to a point

$x = 0, y = \bar{y}, \psi = 0$. How?

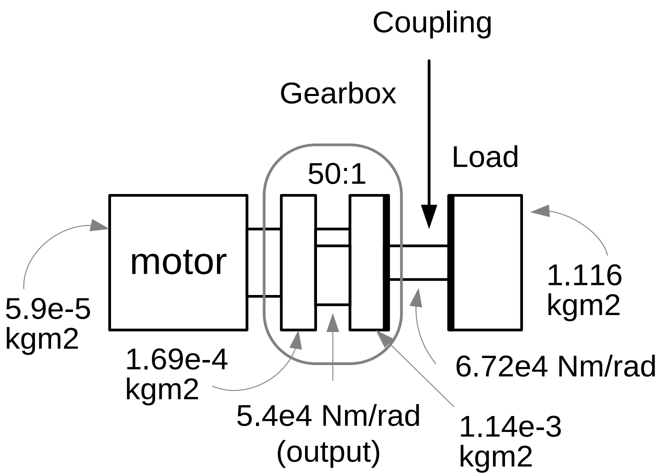
Try this: $u_1(t) = \cos\left(\frac{\pi t}{\text{[illegible]}}\right), u_2(t) = \sin\left(\frac{\pi t}{\text{[illegible]}}\right)$

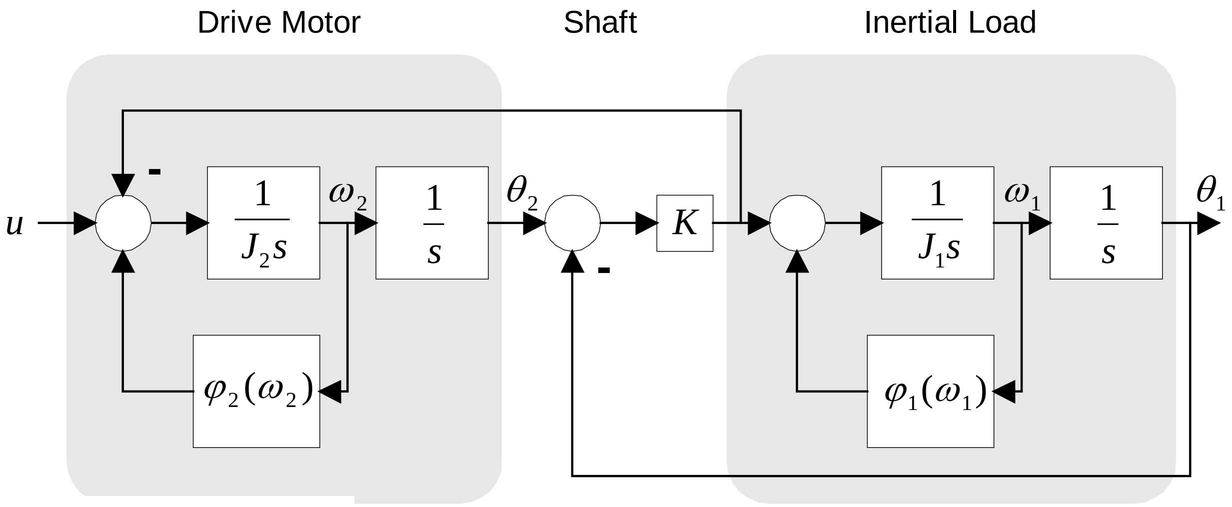


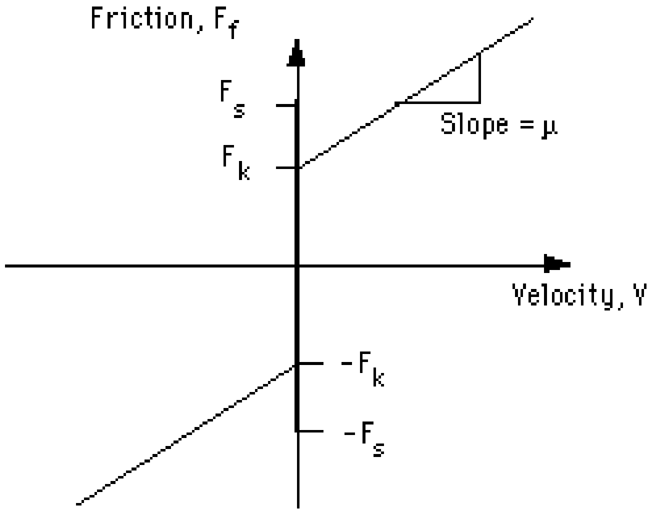
The system appears not to be linearly uncontrollable but nonlinearly controllable!!

By coordinating the rudder and forward speed, we can cause the vehicle to move along the y-axis.

Example 3: Drive Motor & Load with Non-smooth Friction

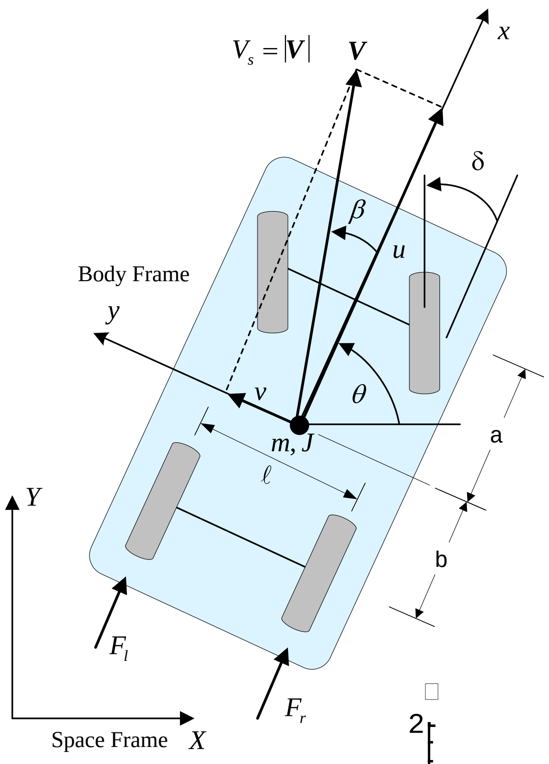






- The problem is to accurately position the load.
- The problem becomes even more interesting if the friction parameters are uncertain.

Example 4: Automobile – Multiple Equilibria

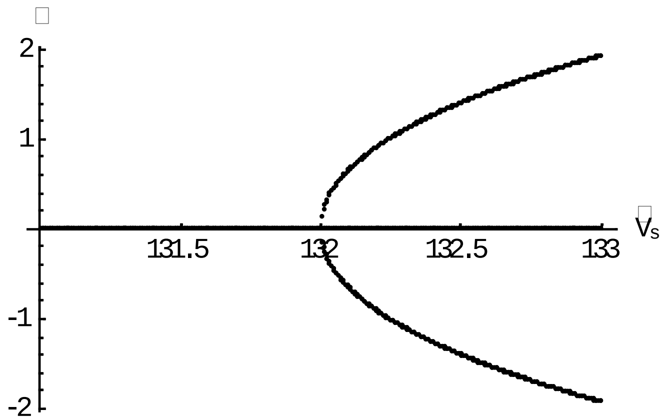


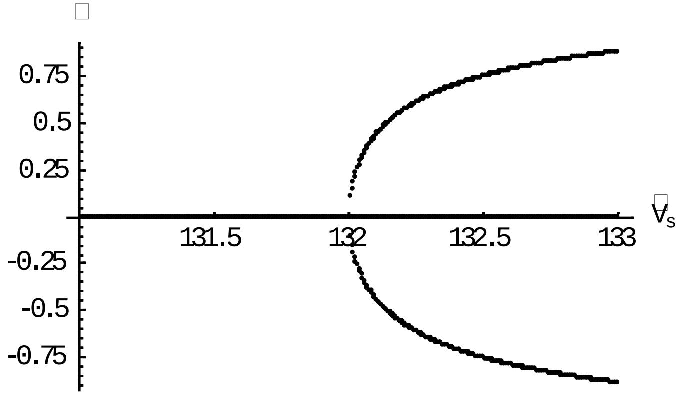
$$\frac{d}{dt}\begin{bmatrix}\omega \\ V_s \\ \beta\end{bmatrix} = F\left(\omega, V_s, \beta, \delta, F_d\right)$$

equilibria: set $\delta = 0, V_s = \bar{V}_s$ (a parameter)

solve for ω, β, F_d

$$0 = F\left(\omega, \bar{V}_s, \beta, 0, F_d\right)$$





Example 5 – Simple 1 dof

$$\dot{\theta} = \omega$$

$$\dot{\omega} = u$$

replace θ by $x, y \rightarrow \begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$

$$\dot{x} = -y\omega$$

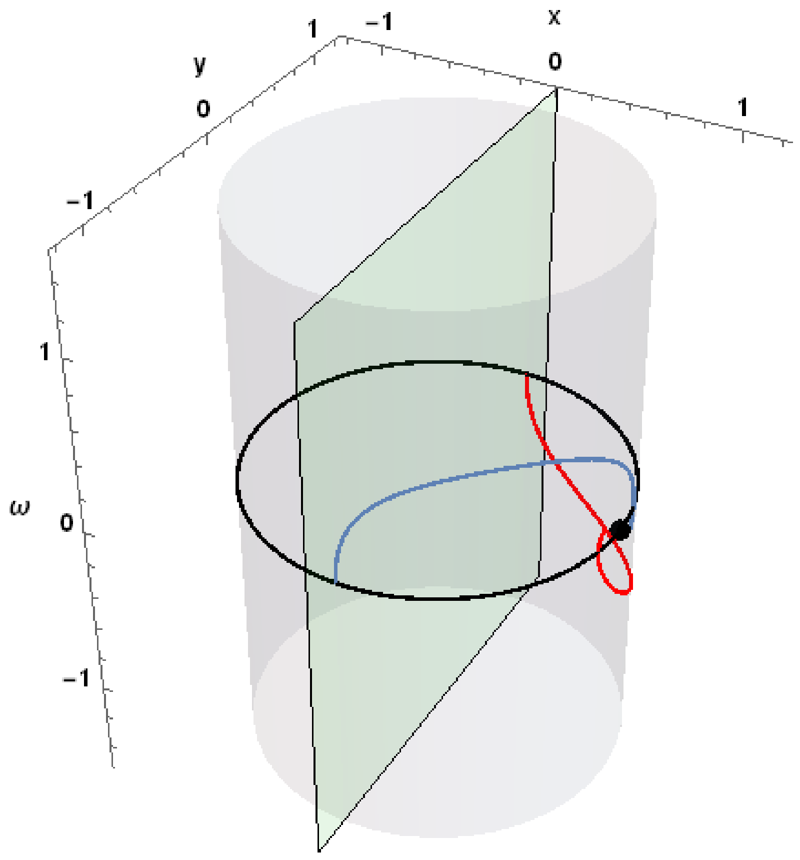
$$\dot{y} = x\omega$$

$$\dot{\omega} = u$$

$$x^2 + y^2 = 1$$

A stabilizing controller is easily obtained via

Lyapunov design $u = -y - \omega \Rightarrow u = -\sin\theta - \omega$



Example 6 - GTM

$$\dot{\theta} = q$$

$$\dot{x} = V\cos\gamma$$

$$\dot{z} = V\sin\gamma$$

$$\dot{V} = \frac{1}{m}\left(T\cos\alpha - \tfrac{1}{2}\rho V^2 S C_D\left(\alpha,\delta_e,q\right) - mg\sin\gamma\right)$$

$$\dot{\gamma} = \frac{1}{mV}\left(T\sin\alpha + \tfrac{1}{2}\rho V^2 S C_L\left(\alpha,\delta_e,q\right) - mg\cos\gamma\right)$$

$$\dot{q} = \frac{M}{I_y}, \quad M = \left(\tfrac{1}{2}\rho V^2 S\bar{c}C_m\left(\alpha,\delta_e,q\right) + \tfrac{1}{2}\rho V^2 S\bar{c}C_Z\left(\alpha,\delta_e,q\right)\left(x_{cgref} - x_{cg}\right) - mg\,x_{cg} + l_t T\right)$$

$$\alpha = \theta - \gamma$$

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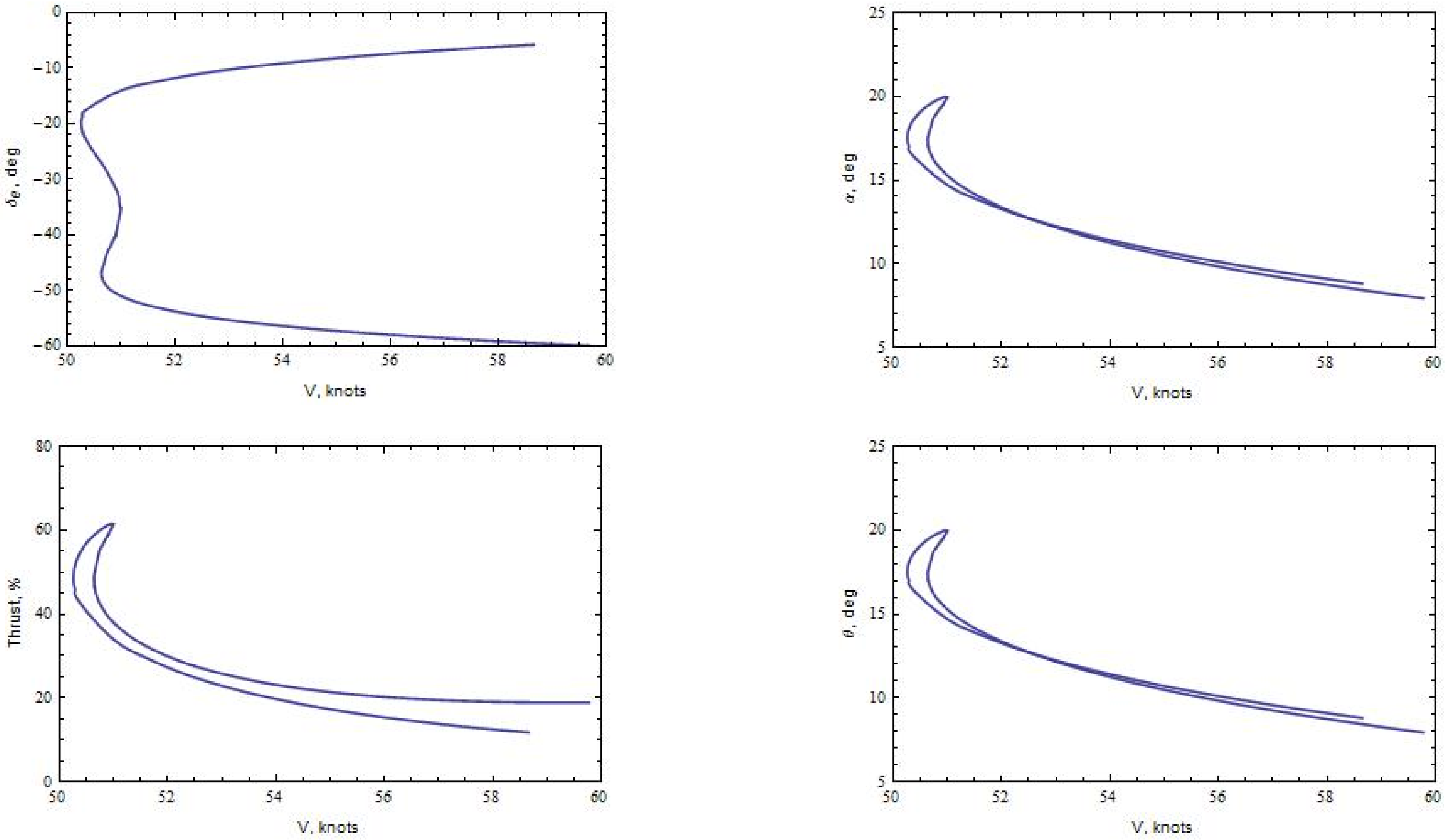
GTM Equilibrium Surface

Straight and Level Flight Analysis



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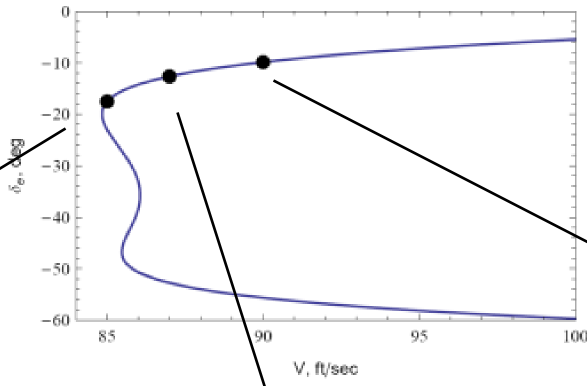
Nonlinear Equilibrium Structure Analysis of Upset Using GTM (1)

Coordinated Turn Analysis



Coordinated turn of GTM @ 85 fps

Coordinated turn of GTM @ 87 fps

Coordinated turn of GTM @ 90 fps

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Nonlinear Equilibrium Structure Analysis of Upset Using GTM (2)

Coordinated Turn Analysis