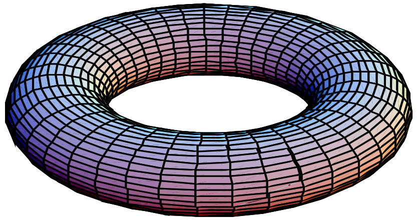
Switching & Hybrid Systems

Harry G. Kwatny

Department of Mechanical Engineering & Mechanics

Drexel University



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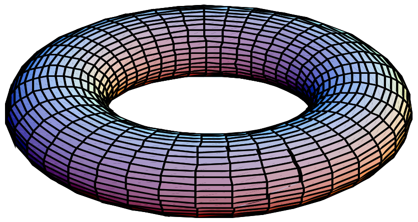
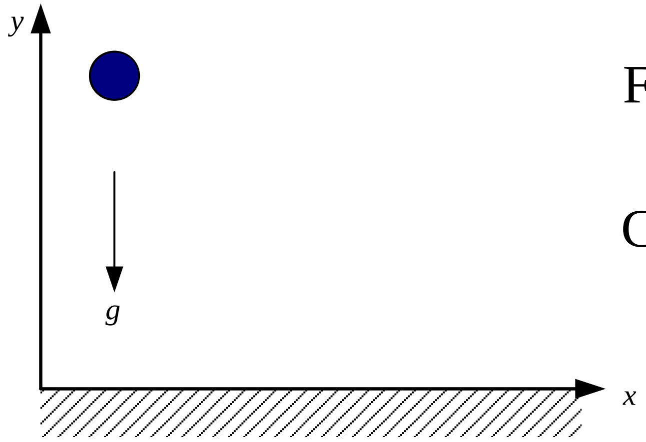
Outline

- Simple Examples of Hybrid Systems
  - Bouncing ball
  - Heating system
  - Gearbox/cruise control
- Simulating Hybrid Systems – Simulink with Stateflow
  - Stateflow/Simulink
  - Bouncing ball
  - Power conditioning system
  - Inverted pendulum
- Hybrid System Models
  - Hybrid automaton
  - IP formulas

Simple Examples



Bouncing Ball

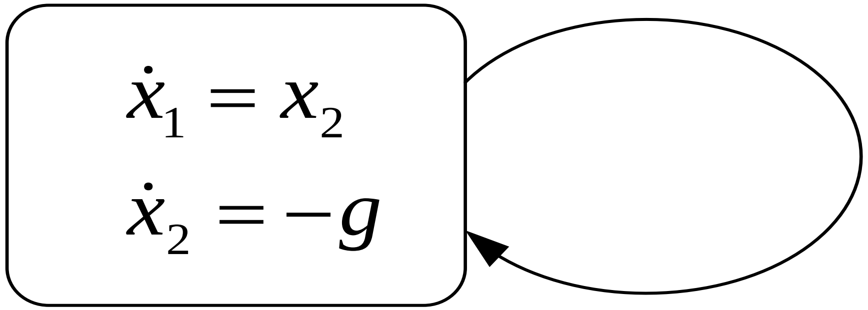


Free fall: $\ddot{y} = -g$

Collision:

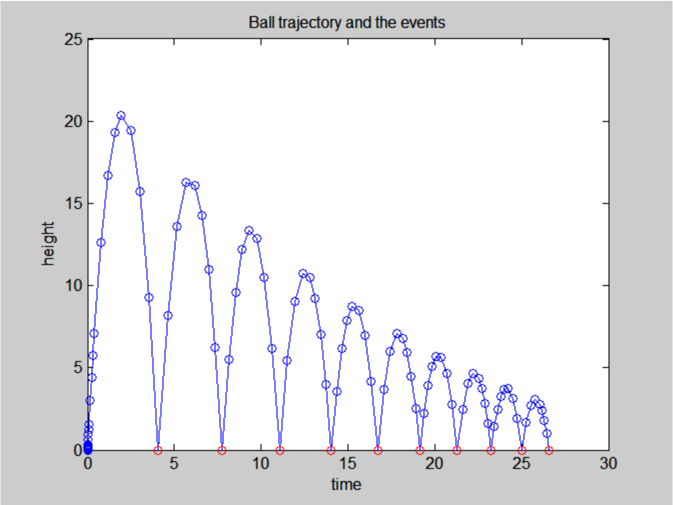
$$y(t^+) = y(t^-) = 0$$

$$\dot{y}(t^+) = -c\dot{y}(t^-),\ c \in [0,1]$$



$$x_1 = 0 \wedge x_2 < 0 \Rightarrow x_2^+ := -cx_2^-$$

Bouncing Ball



$$v = v_0 - gt$$

$$x = v_0 t - \frac{1}{2} g t^2 = 0 \Rightarrow t = \frac{2v_0}{g}$$

$$t_1 = \frac{2v_0}{g}, t_2 = \frac{2v_0}{g} c, \ldots, t_i = \frac{2v_0}{g} c^{i-1}$$

$$T_N = \sum_{i=1}^{N} t_i = \sum_{i=1}^{N} \frac{2v_0}{g} c^{i-1} = \frac{2v_0}{g} \sum_{i=1}^{N} c^{i-1} = \frac{2v_0}{g} \left[\frac{1 - c^N}{1 - c} \right]$$

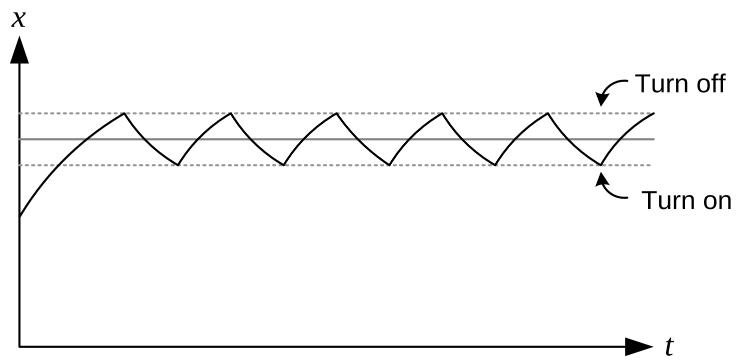
Time for N transitions

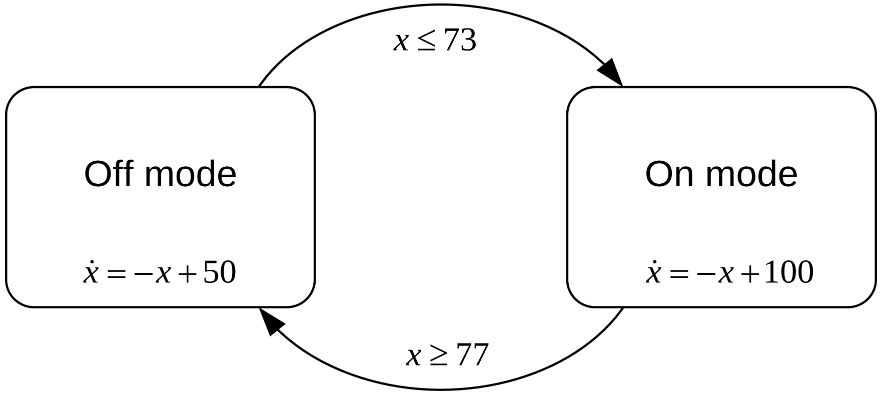
$$\text{For } 0 \le c < 1, \lim_{N \to \infty} T_N = \frac{2v_0}{g} \left[\frac{1}{1 - c} \right]$$

$$\Updownarrow$$

Zeno phenomenon ∞ transitions in finite time

Heater





continuous state: $x \in R$,

$$\dot{x} = f(x,q), \quad f(x,q) = \begin{cases} -x+50 & q = off \\ -x+100 & q = on \end{cases}$$

discrete state: $q \in \{on, off\}$,

$$q(k+1) = \varphi(x, q(k)), \quad \varphi(x,q) = \begin{cases} on & q = off, x \le 73 \\ off & q = off, x > 73 \\ off & q = on, x \ge 77 \\ on & q = on, x < 77 \end{cases}$$

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Automobile Gearbox Control

- Problem 1: Automated gearbox - coordinate automatic gearshift with throttle command
- Problem 2: Cruise control – automate throttle and gearbox to maintain speed
- Background
  - R. W. Brockett, "Hybrid Models for Motion Control," in *Perspectives in Control*, H. L. Trentelman and J. C. Willems, Ed. Boston: Birkhauser, 1993, pp. 29-54.
  - S. Hedlund and A. Rantzer, "Optimal Control of Hybrid Systems," presented at Conference on Decision and Control, Phoenix, AZ, pp. 3972-3977, 1999.
  - F. D. Torrisi and A. Bemporad, "HYSDEL-A Tool for Generating Computational Hybrid Models for Analysis and Synthesis Problems," *IEEE Transactions on Control Systems Technology*, vol. 12, pp. 235-249, 2004.

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Transmission

$$M_{q_i}\dot{v} = \frac{R_{q_i} R_{fin}}{r_{wheel}} f_{eng}(\omega)u - F_b - cv^2 - M_{veh} g \sin(\alpha)$$

$u \in [0,1]$	throttle position
ω	engine speed
$f_{eng}(\omega)u$	engine torque-speed characteristic
$q \in \{q_1, q_2, q_3, q_4, q_5\}$	transmission state
$R_{q_i}, i = 1,\ldots,5$	corresponding gear ratios
R_{fin}	rear gear ratio
r_{wheel}	wheel radius
F_b	brake force

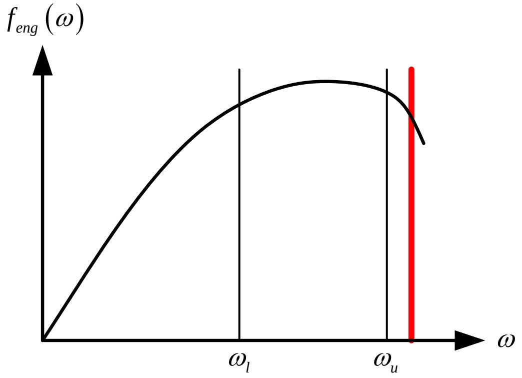
Cruise Control

- Continuous control - throttle u and brake F_b are chosen so that

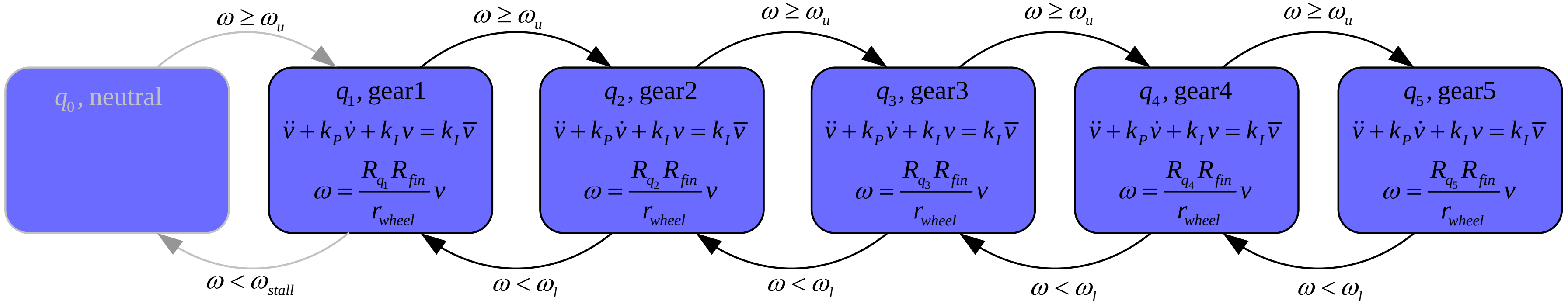
$$\frac{R_{q_i} R_{fin}}{r_{wheel}} f_{eng}(\omega) u - F_b = cv^2 + M_{veh} g \sin(\alpha) + M_{q_i}\left[k_P(\bar{v} - v) + k_I \int (\bar{v} - v)\, dt\right]$$

- a standard feedback linearized design with PI controller.

- notice that control depends on the discrete state.



- Discrete control - ad hoc gear shift strategy.



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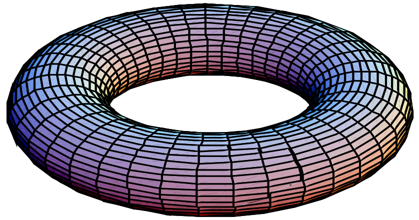
Cruise Control Issues

- Choice of shift thresholds
  - Wide spread implies large speed deviation before shift
  - Narrow spread opens possibility of excessive shifting, even chattering
- Does not explicitly consider throttle and brake limits
- It must be verified that the engine does not stall or exceed red line

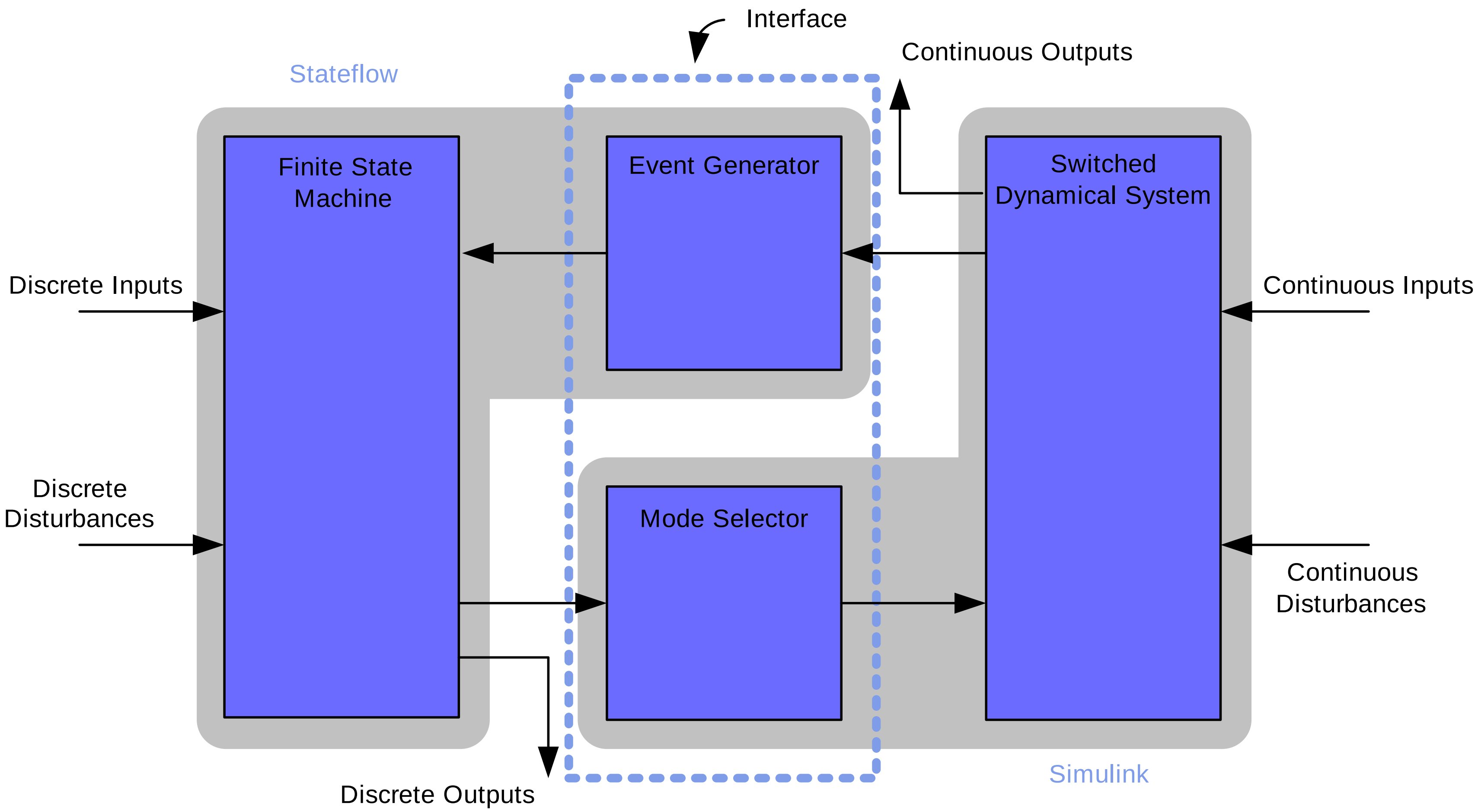
Simulating Hybrid Systems with Stateflow/SIMULINK



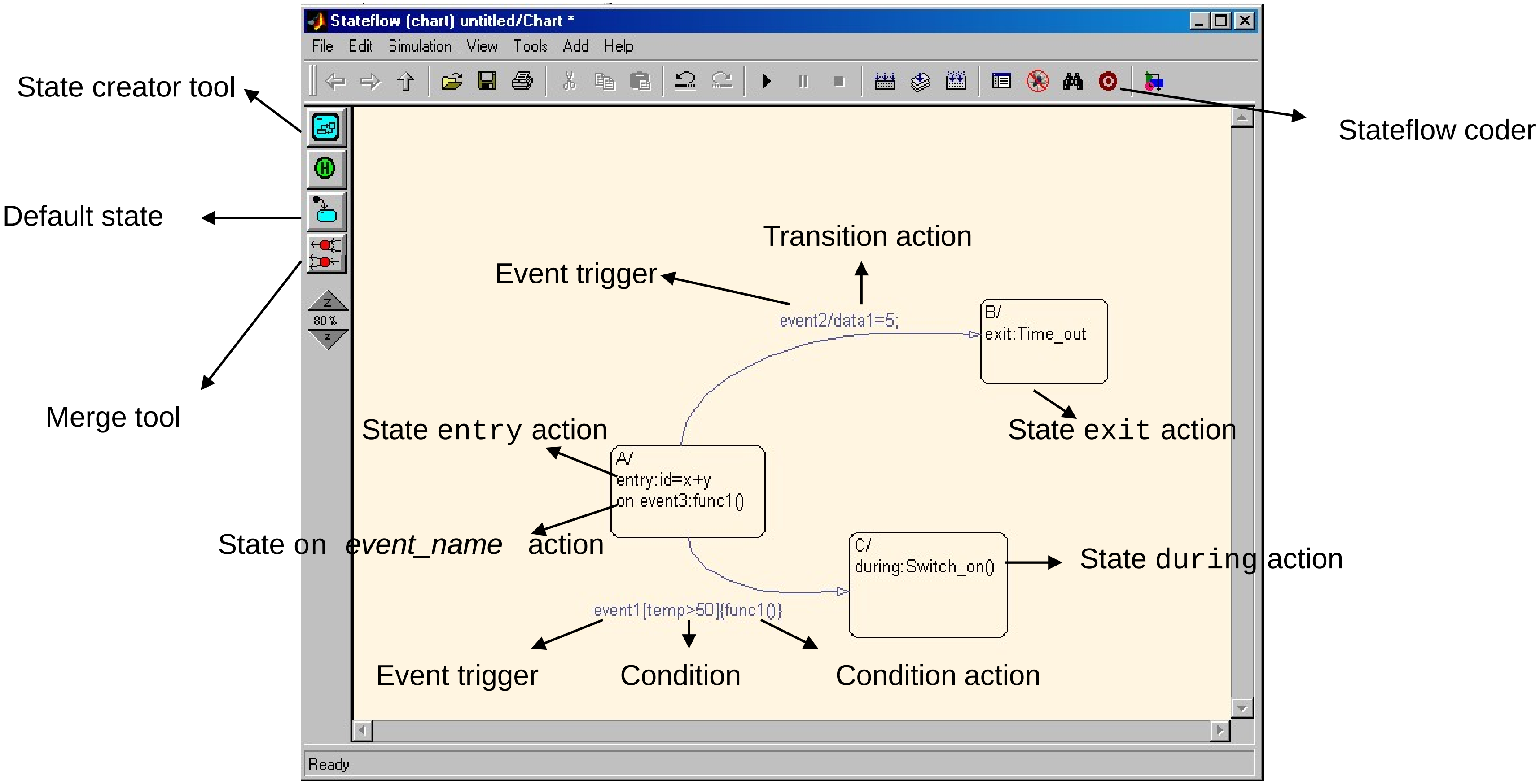
Stateflow

- Stateflow is a Simulink toolbox
- Provides a graphical means to incorporate discrete event process into Simulink
- Based on the concept of statecharts
  - Harel, D., *Statecharts: A Visual Formalism for Complex Systems.* Science of Computer Programming, 1987. **8**: p. 231-274.
  - Has evolved to represent an implementation of UML

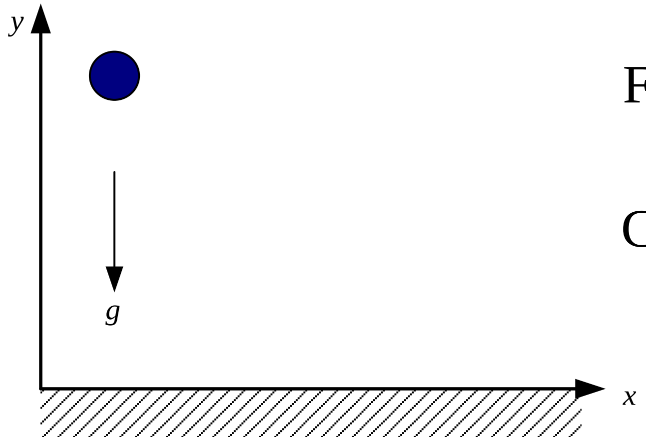
Simulating Hybrid Systems in Stateflow/SIMULINK



Stateflow: Action Language Categories



Bouncing Ball



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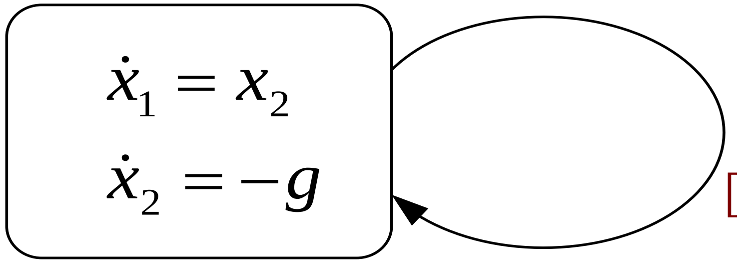
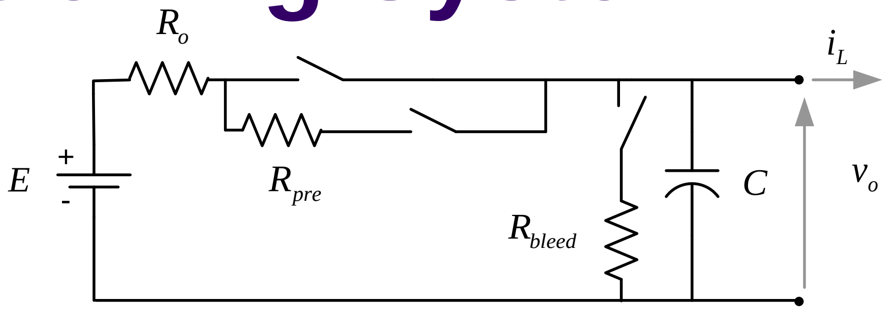


$$x_1 = 0 \wedge x_2 < 0 \Rightarrow x_2^+ := -cx_2^-$$

$$[x_1 = 0 \,\&\&\, x_2 < 0]\{x_2^+ := -cx_2^-\}$$

StateFlow action language

Power Conditioning System



- High power drives in vehicle applications
  - Startup (precharge)
  - Normal (current regulation)
  - Shutdown (bleed)
- Background (Boost converters)
  - M. Senesky, G. Eirea, and T. J. Koo, "Hybrid Modeling and Control of Power Electronics," in *Hybrid Systems: Computation and Control*, vol. 2623, *Lecture Notes in Computer Science*. New York: Springer-Verlag, 2003, pp. 450-465.
  - P. Gupta and A. Patra, "Hybrid Sliding Mode Control of DC-DC Power Converters," presented at IEEE Tencon 2003, Bangalore, 2003.

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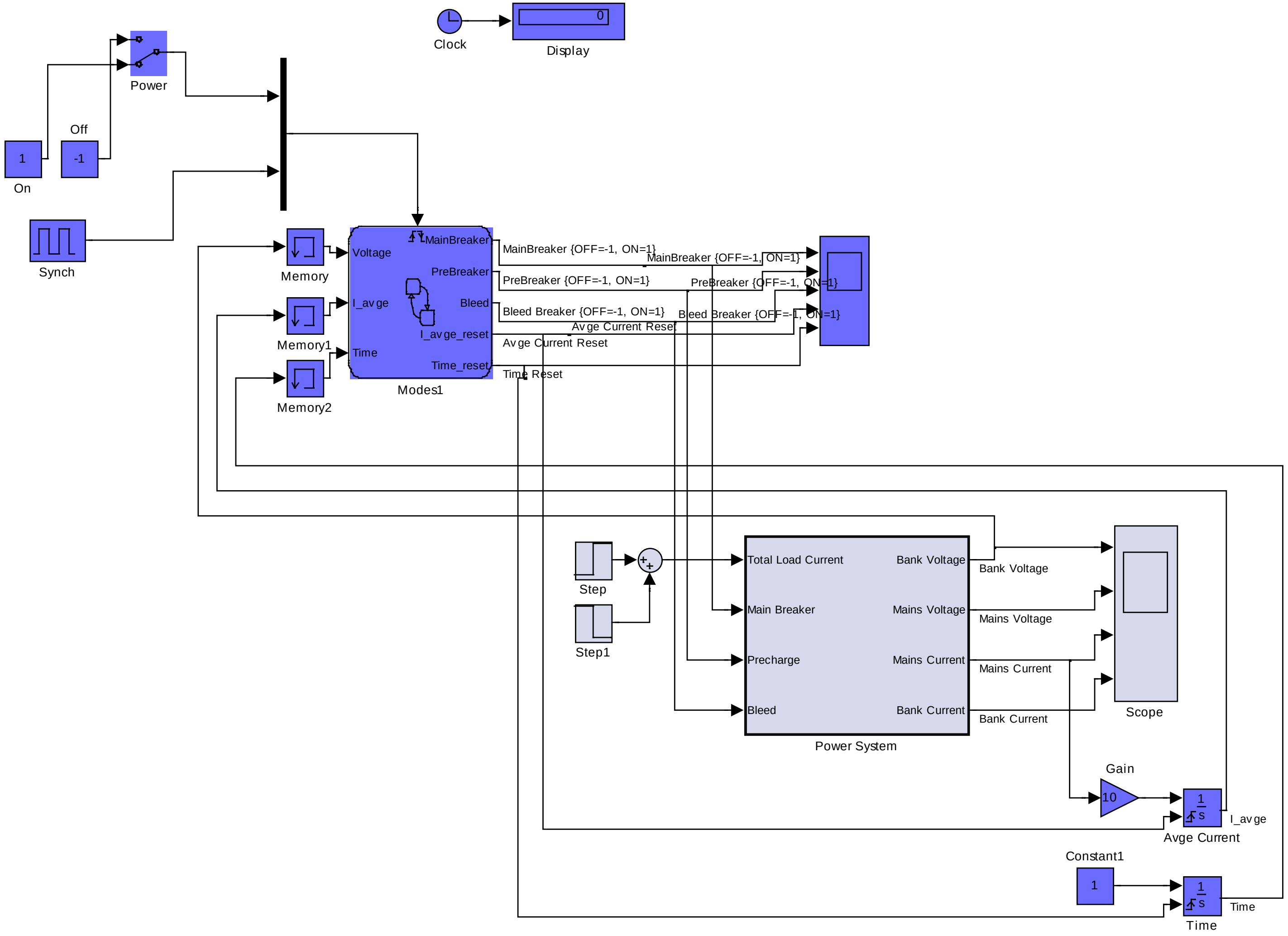
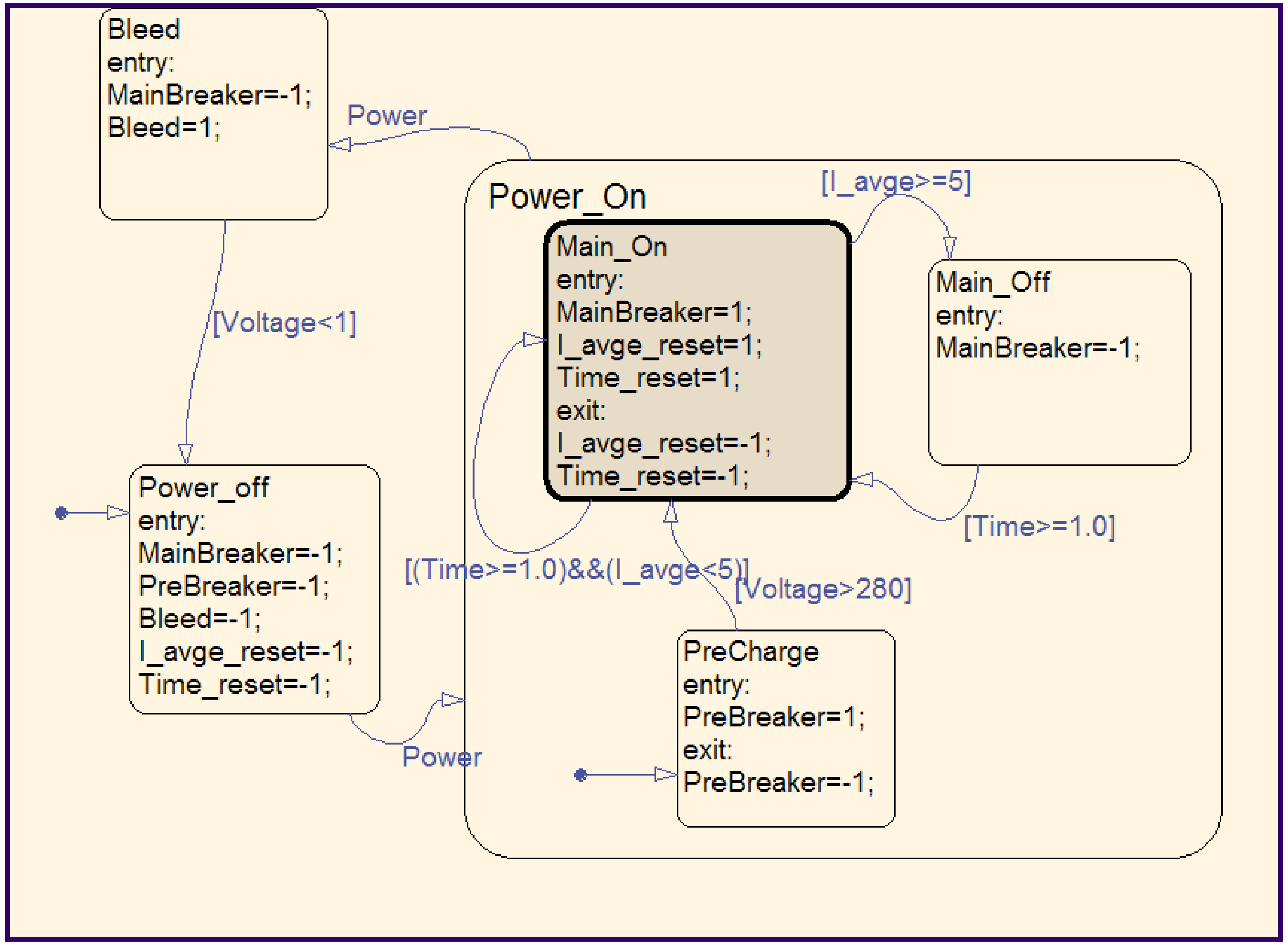
Power Conditioning System



Power Conditioning System



Power Conditioning System