Optimal Control of Hybrid Systems

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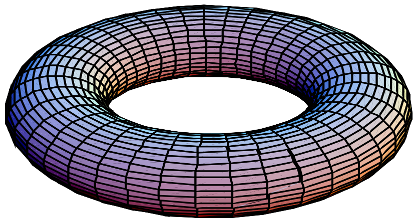

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Outline

- Background
- Dynamic Programming Control Design
- Computations
- Examples
  - 3 Bus
  - Induction Motor with UPS
  - DDG 1000 All Electric Ship
  - Hybrid Electric Drive

Optimal Control of hybrid Systems: Background

- Discrete event systems
  - K. M. Passino and P. J. Antsaklis, "On the Optimal Control of Discrete Event Systems," presented at Conference on Decision and Control, Tampa, FL, pp. 2713-2718, 1989.
  - R. Kumar and V. Garg, "Optimal Supervisory Control of Discrete Event Dynamical Systems," *SIAM Journal of Control and Optimization*, vol. 33, pp. 419-439, 1995.
  - R. Sengupta and S. Lafortune, "An Optimal Control Theory for Discrete Event Systems," *SIAM Journal of Control and Optimization*, vol. 36, pp. 488-451, 1998.
- Hybrid systems (deterministic)
  - M. S. Branicky, V. S. Borkar, and S. K. Mitter, "A Unified Framework for Hybrid Control: Model and Optimal Control Theory," *IEEE Transactions on Automatic Control*, vol. 43, pp. 31-45, 1998.
  - H. J. Sussmann, "A maximum principle for hybrid optimal control problems," presented at Conference on Decision and Control, Phoenix, AZ, pp. 425-430, 1999.
  - A. Bemporad and M. Morari, "Control of Systems Integrating Logic, Dynamics, and Constraints," *Automatica*, vol. 35, pp. 402-427, 1999.
  - C. Cassandras, D. Pepyne, and Y. Wardi, "Optimal Control of a Class of Hybrid Systems," *IEEE Transactions on Automatic Control*, vol. 46, pp. 398-415, 2001.
  - M. Alimar and S.-H. Attia, "On Solving Optimal Control Problems for Switched Hybrid Nonlinear Systems by Strong variations Algorithm," presented at NOLCOS 04, 2004.

Control System Design: Finite Horizon Optimization

- We seek controls that are optimal when viewed over a finite time period
  - Optimal control typically varies over the time period
  - Moving Horizon - sometimes referred to as model predictive control, receding horizon control, or finite look-ahead control
  - Periodic – reinitialize control when final time is reached
- We want feedback controls
  - The feedback control should be explicitly computed off-line
- In the linear case with quadratic cost it is known that receding horizon feedback controls can stabilize a desired equilibrium

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Control via Dynamic Programming: the principle of optimality

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k))$$

The Principle of Optimality: if a trajectory beginning at $k = 0$ is optimal, then that portion of the trajectory beginning at any $k = i, 0 \le i \le N-1$ is optimal.

$J_i^*(x_i)$ denotes the optimal cost of a trajectory beginning in state x_i at time $k = i$. Then

$$J_{i-1}^*(x_{i-1}) = \min_{\mu_{i-1}} \left\{ g_{i-1}(x_{i-1}, \mu_{i-1}(x_{i-1})) + J_i^*(x_i) \right\}$$

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Problem Definition

$$x_{k+1} = f\left(x_k, \delta_k, d_k, z_k, u_k\right), \quad k = 0,1,\ldots,N-1$$

$$h\left(x_k, \delta_k, d_k, z_k, u_k, \delta_{k-1}, d_{k-1}, z_{k-1}\right) \le 0$$

x_k the continuous state (real numbers)

δ_k the discrete state or "mode" (binary numbers)

u_k the control, may be composed of discrete and continuous elements

d_k discrete (binary) auxiliary variables

z_k continuous (real) auxiliary variables

A control policy is: $\pi = \left\{\mu_0\left(x_0,\delta_0\right), \mu_1\left(x_1,\delta_1\right), \ldots, \mu_{N-1}\left(x_{N-1},\delta_{N-1}\right)\right\}$, such that $u_k = \mu_k\left(x_k,\delta_k\right)$

The optimal policy ninimizes the cost:

$$J_\pi\left(x_0,\delta_0\right) = g_N\left(x_N,\delta_N\right) + \sum_{k=0}^{N-1} g_k\left(x_k, \delta_k, \mu_k\left(x_k,\delta_k\right)\right)$$

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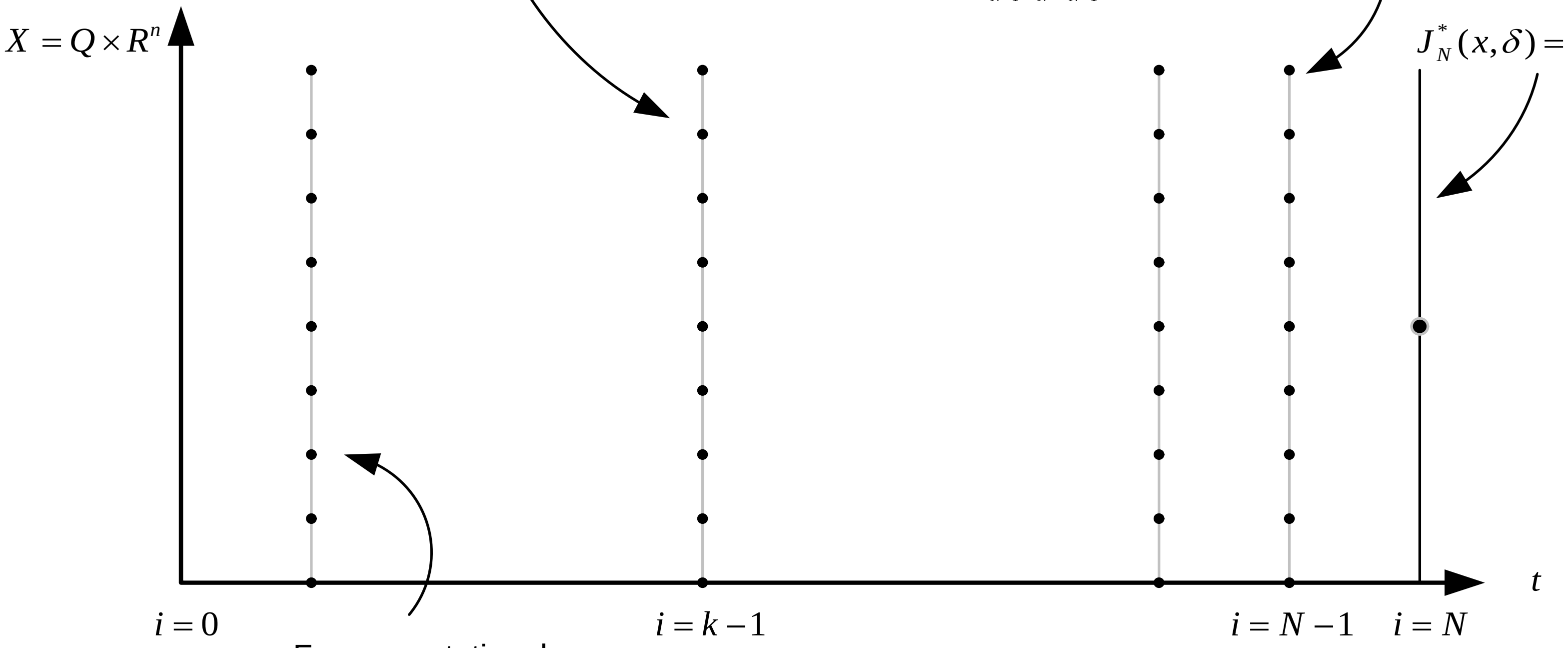
Summary of DP Computation

From each state at i=N-1 compute the optimal control for this stage. The optimization is carried out with constraints: mixed integer inequalities and dynamics.

$$J^*_{k-1}\left(x_{k-1},\delta_{k-1}\right)=\min_{\substack{u_{k-1}\left(x_{k-1},\delta_{k-1}\right)\\ u_{k-1},x_k,\delta_{k-1}\in C}}\left\{g_{k-1}\left(x_{k-1},\delta_{k-1},u_{k-1}\right)+J^*_k\left(x_k,\delta_k\right)\right\}$$

$$J^*_{N-1}\left(x_{N-1},\delta_{N-1}\right)=\min_{\substack{u_{N-1}\left(x_{N-1},\delta_{N-1}\right)\\ u_{N-1},x_N,\delta_{N-1}\in C}}\left\{g_{N-1}\left(x_{N-1},\delta_{N-1},u_{N-1}\right)+J^*_N\left(x_N,\delta_N\right)\right\}$$

$$J^*_N\left(x,\delta\right)=g_N\left(x,\delta\right)$$

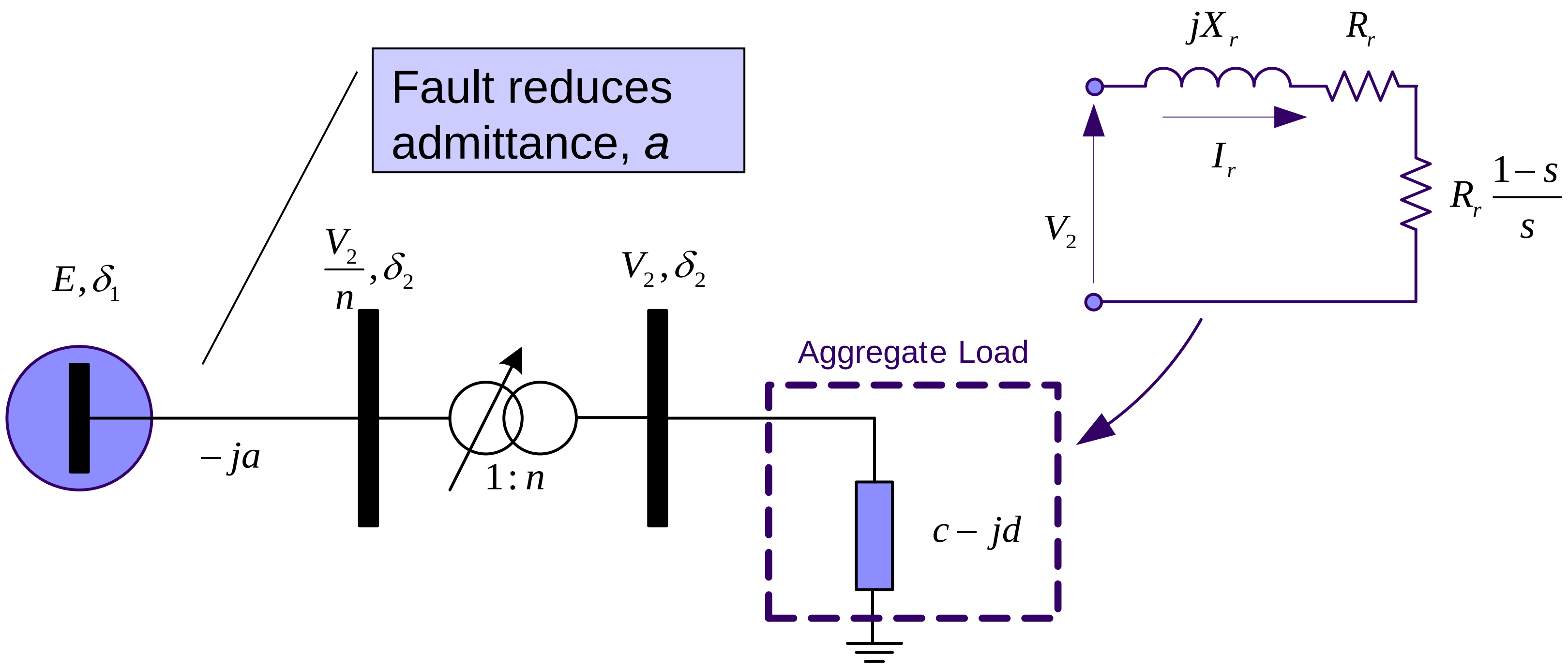


For computational purposes discretize the state space.

Summary of DP Algorithm

- Separate the inequalities into binary and real sets
  - binary equations contain only binary variables, real equations can contain both binary and real variables.
- Use the *Mathematica* function Reduce to obtain all feasible solutions of the binary inequalities.
  - if there are N binary variables then there are 2^N combinations to be evaluated if one were to attempt to optimize by enumeration. Reduce identifies the few feasible solutions very rapidly.
- Use Reduce to solve the real inequalities for the real variables for every feasible combination of binary variables.
  - Many of these combinations of binary variables will not admit feasible real variables, so they can be dropped.
- Enumerate the values of the cost for each feasible pair of binary and real variables and select the minimum.

3-Bus Example -Aggregate Load with Induction Motors



• H. Ohtsuki, A. Yokoyama, and Y. Sekine, "Reverse Action On-Load Tap Changer in Association with Voltage Collapse," *IEEE Tansactions on Power Systems*, vol. 6, pp. 300-306, 1991.

• M. K. Pal, "Voltage Stability: Analysis Needs, Modelling requirement, and Modelling Adequacy," *IEE Proceedings - C*, vol. 140, pp. 279-286, 1993.

• L. Bao, X. Duan, and T. He, "Analysis of Voltage Collapse Mechanisms in State Space," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 147, pp. 395-400, 2000.

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3-Bus Cont’d

Optimal control problem (case: tap fixed $n = 1$)

Determine

- field voltage $E \in [0, 2]$ and

- amount of load shedding $\eta \in \{0, 0.4, 0.8\}$ needed to keep load bus voltage V_2 close to 1.

- no cost is attributed to E, so $(V_2 = 1 \wedge 0 < E < 2) \vee (E = 2)$

- cost is attributed to dropping load, so choose η to minimize

$$J = \sum_{k=0}^{N-1} \left(\left\| V_2(k) - 1 \right\|^2 + r_1 \left\| \eta_L(k) \right\|^2 \right)$$

System equations:

$$V_2 = \frac{a/n}{\sqrt{c^2 + d^2}} E, \quad c = (1-\eta)\left(\frac{1}{R_L} + \frac{R_r s}{R_r^2 + s^2 X_r^2} \right), \quad d = (1-\eta)\left(\frac{X_r s^2}{R_r^2 + s^2 X_r^2} \right)$$

$$\dot{s} = \frac{(1-\eta)}{I_m \omega_0^2} \left(P_m - V_2^2 \frac{R_r s (1-s)}{R_r^2 + s^2 X_r^2} \right)$$

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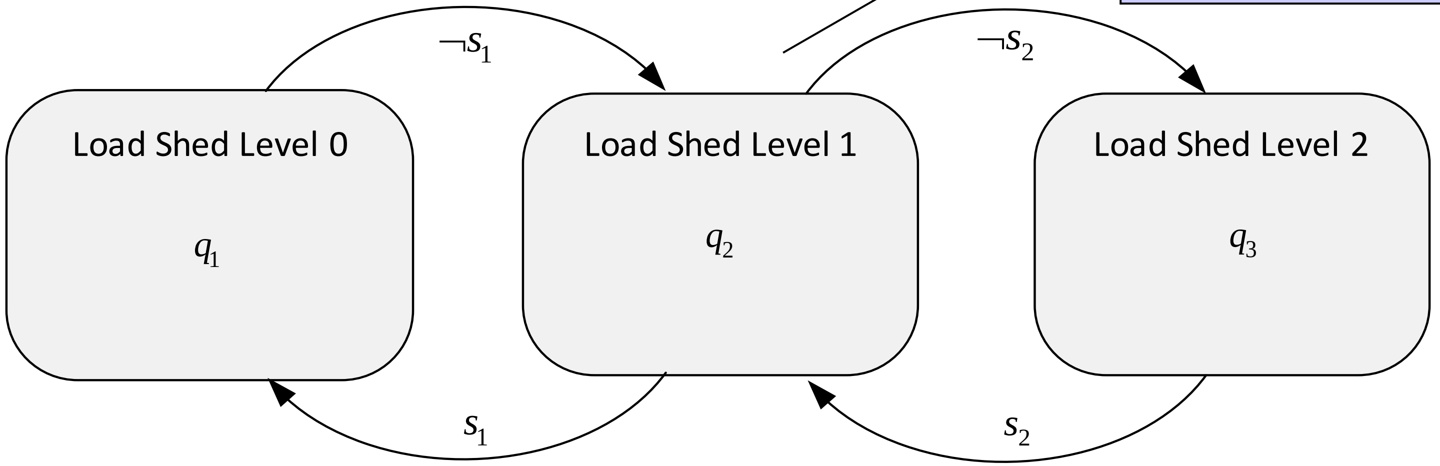
Transition Dynamics

Can impose dwell time



$$L = exactly\left(1,\left\{q_1(t),q_2(t),q_3(t)\right\}\right) \wedge exactly\left(1,\left\{q_1(t^+),q_2(t^+),q_3(t^+)\right\}\right) \wedge$$

$$\left(q_1(t) \wedge \neg s_1 \Rightarrow q_2(t^+)\right) \wedge \left(q_1(t) \wedge s_1 \Rightarrow q_1(t^+)\right) \wedge$$

$$\left(q_2(t) \wedge \neg s_2 \Rightarrow q_3(t^+)\right) \wedge \left(q_2(t) \wedge s_1 \Rightarrow q_1(t^+)\right) \wedge \left(q_2(t) \wedge \neg\left(s_1 \vee \neg s_2\right) \Rightarrow q_2(t^+)\right) \wedge$$

$$\left(q_3(t) \wedge s_2 \Rightarrow q_2(t^+)\right) \wedge \left(q_3(t) \wedge \neg s_2 \Rightarrow q_3(t^+)\right)$$

IP Formulas: Transition Dynamics

$$1-\delta_{q_1}-\delta_{q_2}-\delta_{q_3}\geq 0,\quad -1+\delta_{q_1}+\delta_{q_2}+\delta_{q_3}\geq 0$$

$$1-\delta_{q_1^+}-\delta_{q_2^+}-\delta_{q_3^+}\geq 0,\quad -1+\delta_{q_1^+}+\delta_{q_2^+}+\delta_{q_3^+}\geq 0$$

$$1-\delta_{q_1}+\delta_{q_1^+}-\delta_{s_1}\geq 0,\quad 1-\delta_{q_2}+\delta_{q_1^+}-\delta_{s_1}\geq 0$$

$$1-\delta_{q_2}+\delta_{q_2^+}-\delta_{s_2}\geq 0,\quad 1-\delta_{q_3}+\delta_{q_2^+}-\delta_{s_2}\geq 0$$

$$-\delta_{q_1}+\delta_{q_2^+}+\delta_{s_1}\geq 0$$

$$-\delta_{q_2}+\delta_{q_3^+}+\delta_{s_2}\geq 0,\quad -\delta_{q_3}+\delta_{q_3^+}+\delta_{s_2}\geq 0$$

$$0\leq\delta_{q_1}\leq 1, 0\leq\delta_{q_2}\leq 1, 0\leq\delta_{q_3}\leq 1$$

$$0\leq\delta_{q_1^+}\leq 1, 0\leq\delta_{q_2^+}\leq 1, 0\leq\delta_{q_3^+}\leq 1$$

$$0\leq\delta_{s_1}\leq 1, 0\leq\delta_{s_2}\leq 1$$

IP Formulas: Optimization Logical Constraint

Excitation

$$\mathscr{L} = (V_2 = 1 \wedge 0 < E < 2) \vee (E = 2)$$

$$3 - d_1 - E > 0, \quad 1 - d_1 + E > 0, \quad -2d_2 + E \geq 0$$

$$-2d_1 + V_2 \geq 0, \quad -2 + d_1 + V_2 \leq 0$$

$$0 \leq d_1, d_2 \leq 1, \quad 0 \leq E, V_2 \leq 2$$

$$V_2 = \frac{a/n}{\sqrt{c^2 + d^2}} E$$

Network equation

IP Formulas: Load Shed Parameter

$$\mathscr{L} = (q_1^+ \Rightarrow \eta == 0) \wedge (q_2^+ \Rightarrow \eta == 0.4) \wedge (q_3^+ \Rightarrow \eta == 0.8)$$

$$-0.4d_4 + \eta \geq 0 \quad -0.8d_5 + \eta \geq 0$$

$$-1 + d_3 + \eta \leq 0 \quad -1 + 0.6d_4 + \eta \leq 0 \quad -1 + 0.2d_5 + \eta \leq 0$$

$$d_3 - \delta_{q_1^+} \geq 0 \quad d_4 - \delta_{q_2^+} \geq 0 \quad d_5 - \delta_{q_3^+} \geq 0$$

$$0 \leq d_3, d_4, d_5 \leq 1$$

$$0 \leq \eta \leq 1$$

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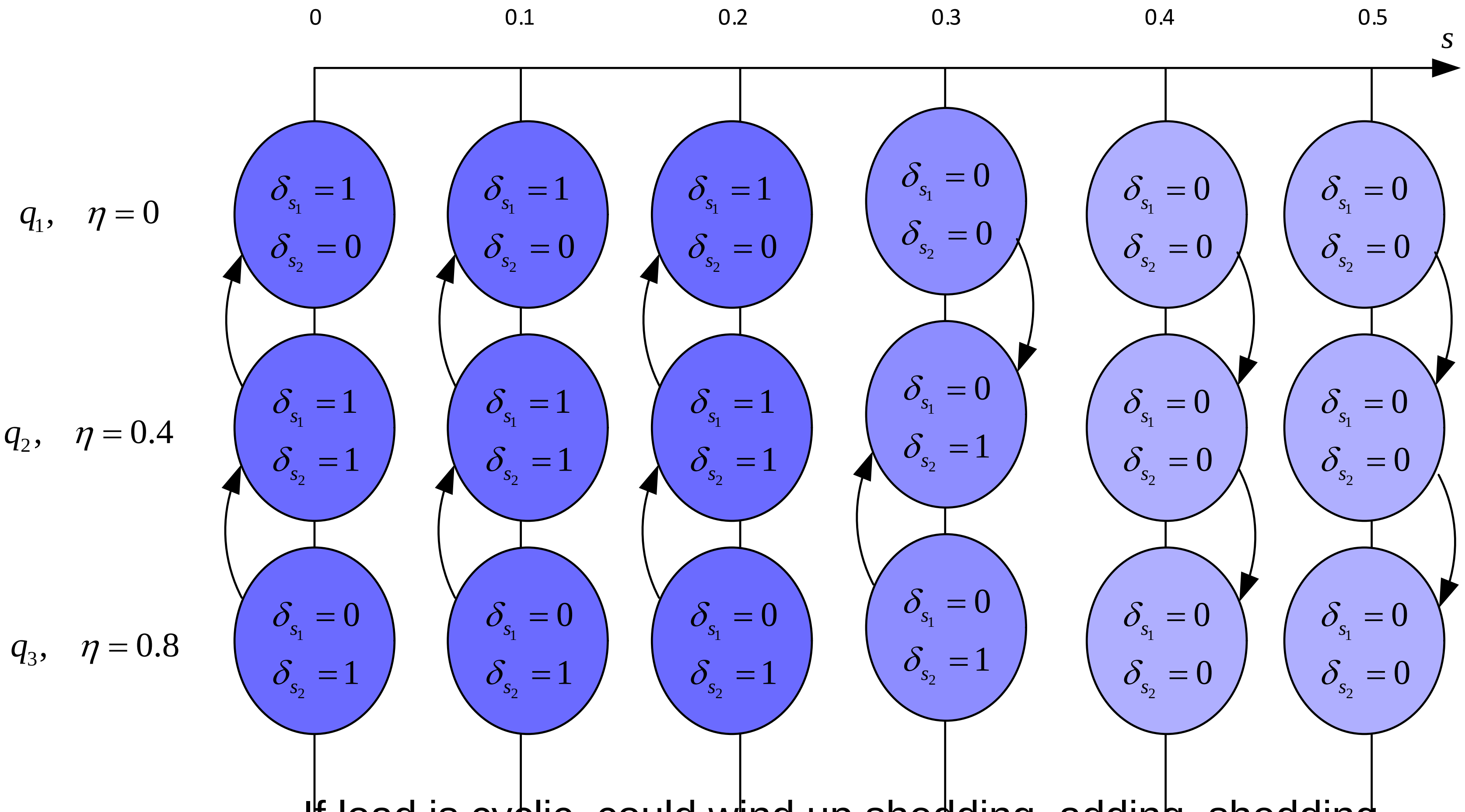
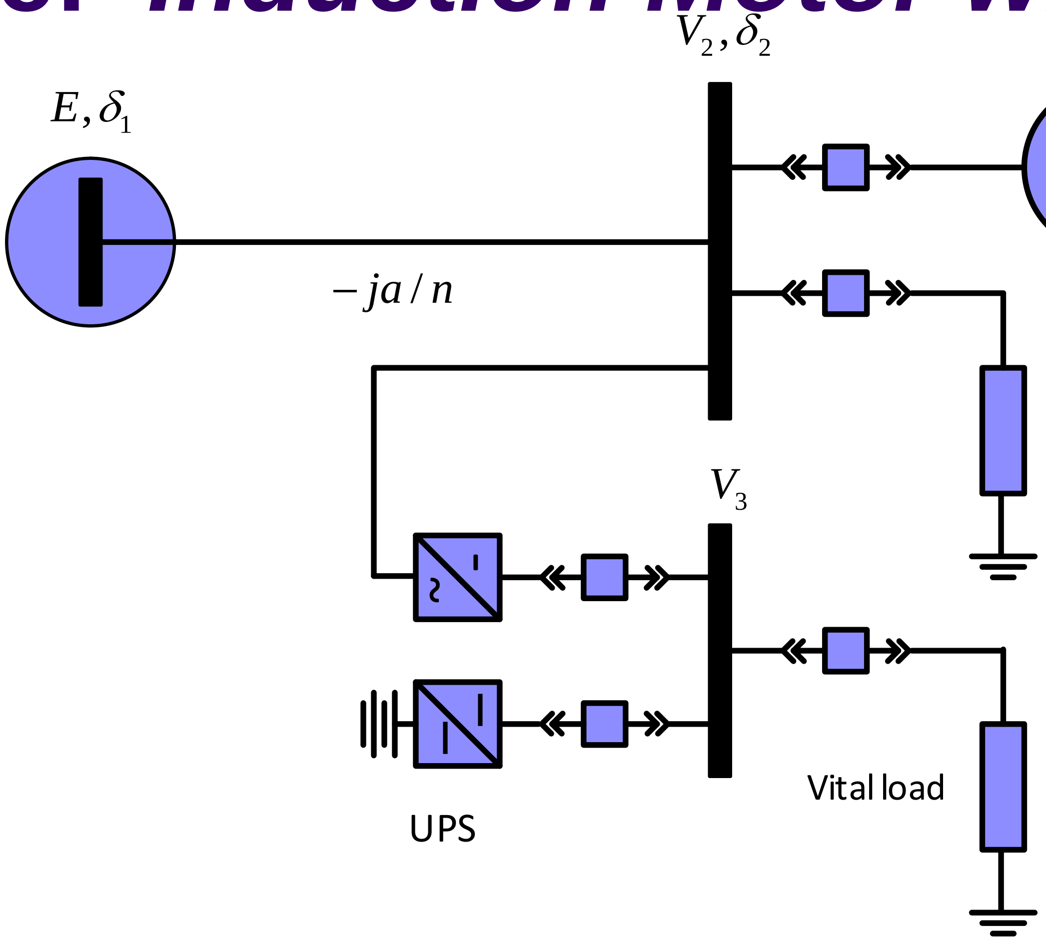
Feedback Policy ~ Post Fault



If load is cyclic, could wind up shedding, adding, shedding,
…One way to minimize this is through transition delay.

Example: *Induction Motor with UPS*



IM-UPS Equations

$$0 = P_2 - b_{12}V_1V_2\sin\theta_2 - g_{22}V_2^2 - g_{23}V_2V_3\cos(\theta_2-\theta_3) - b_{23}V_2V_3\sin(\theta_2-\theta_3)$$

$$0 = P_3 - g_{33}V_3^2 - g_{23}V_2V_3\cos(\theta_2-\theta_3) - b_{23}V_2V_3\sin(\theta_2-\theta_3)$$

network

$$0 = Q_2 + b_{12}V_1V_2\cos\theta_2 + b_{22}V_2^2 + b_{23}V_2V_3\cos(\theta_2-\theta_3) - g_{23}V_2V_3\sin(\theta_2-\theta_3)$$

$$0 = Q_3 + b_{33}V_3^2 + b_{23}V_2V_3\cos(\theta_2-\theta_3) - g_{23}V_2V_3\sin(\theta_2-\theta_3)$$

$$P_2 = -P_m,\ Q_2 = 0,\ P_3 = -P_L,\ Q_3 = -Q_L,\ V_1 = E$$

$$\dot\sigma = 0,\ \text{disconected},\ \dot\sigma = \frac{i_c}{C},\ v_b = f(\sigma),\ 0 \le \sigma \le 1,\ \text{charging},$$

battery

$$\dot\sigma = -\frac{V_3}{CR_v},\ V_3 = \text{const., discharging}$$

$$P_L = (1-\eta_L)P_0(1+\sigma/T+2v),\ Q_L = (1-\eta_L)Q_0(1+\sigma/T+2v)$$

load

+ Induction motor

IM-UPS Control problem

Logical specifications are key to setting up the optimization problem -

Control variables:

continuous: $E(k)$ field voltage $0 < E \leq 2$

discrete: $\eta_{\mathrm{L}}(k)$ amount of load shedding $\eta_L \in \{0, 0.4, 0.8\}$

Performance goals:

$V_2 \in [0.95, 1.05]$ and $V_3 \in [0.9, 1.1]$

Strategy:

(a) choose E to control terminal voltage V_2 impose logical

conditions $(V_2 = 1 \wedge 0 < E < 2) \vee (E = 2)$

(b) choose η_L to minimize

$$J = \sum_{k=0}^{N-1} \left(\left\| V_2(k) - 1 \right\|^2 + \left\| \eta_L(k) \right\|^2 + 10\left(\delta_{V_3^+} + \delta_{V_3^-} \right) \right)$$

$$\delta_{V_3^-} = \begin{cases} 1 & V_3 < 0.95 \\ 0 & V_3 \geq 0.95 \end{cases} \qquad \delta_{V_3^+} = \begin{cases} 1 & V_3 > 1.05 \\ 0 & V_3 \leq 1.05 \end{cases}$$

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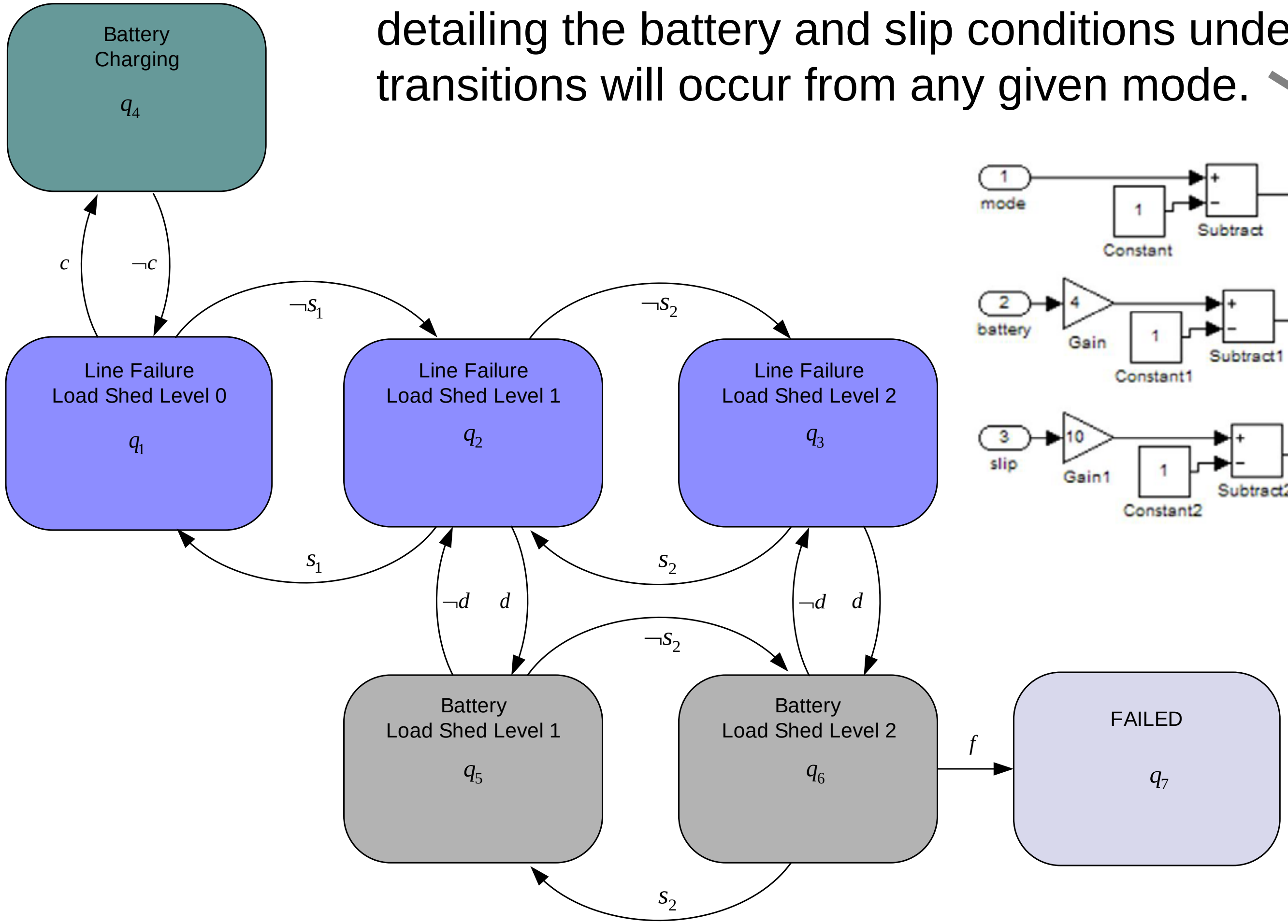
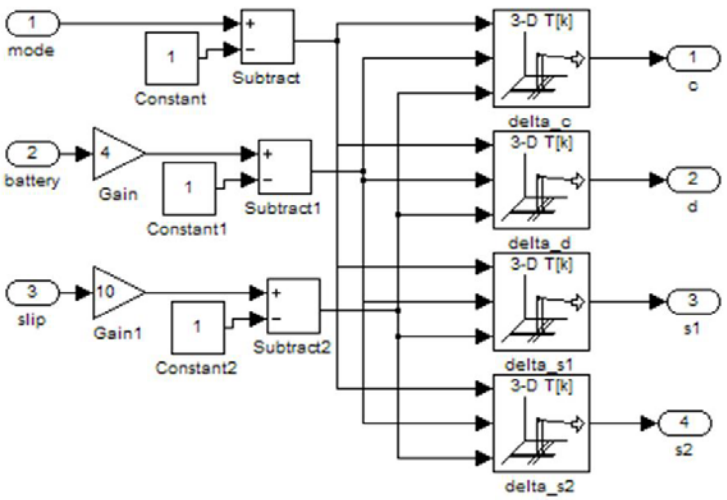
Optimal Control & Transition Diagram

The optimal control takes the form of a lookup table detailing the battery and slip conditions under which transitions will occur from any given mode.

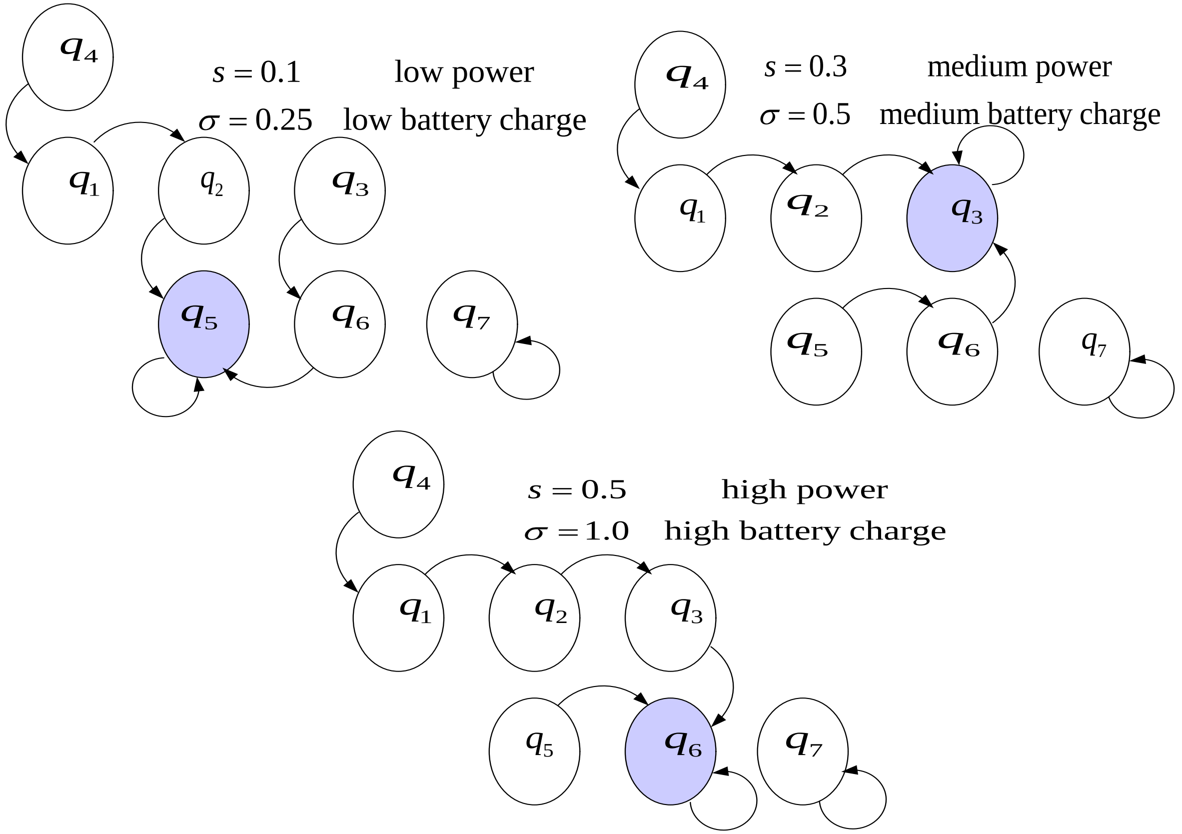




Feedback Policy ~ Post Fault

$s \in \{.1, .2, .3, .4, .5\} \quad \sigma \in \{.25, .5, .75, 1\} \quad q_i \in \{1,2,3,4,5,6,7\},$

Discrete state = 140



The figure shows the optimal switching strategies for 3 selected values of the continuous state.