

MEM 636 THEORY OF NONLINEAR CONTROL - PROBLEM  
SET 1

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**Problem 1.** For each of the scalar systems:

- (1)  $\dot{x} = 2 - 3x + x^2$
- (2)  $\dot{x} = -40x + 40x^2 + 10x^3$

Determine all of the equilibrium points and evaluate their stability.

**Problem 2.** Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1^3 - cx_2\end{aligned}$$

- (1) Show that the origin is an equilibrium point.
- (2) Linearize at the origin and determine if the origin is linearly stable, asymptotically stable.
- (3) Analyze the nonlinear system and determine if the origin is stable, asymptotically stable. If so, estimate the region of attraction.

**Problem 3.** Investigate the stability of the origin, including estimates of the domain of attraction, of the following systems:

- (1)  $\ddot{x} = x - \text{sat}(2x + \dot{x})$ .
- (2)  $\ddot{x} + \dot{x}|\dot{x}| + x - x^3 = 0$ .