MEM 636 THEORY OF NONLINEAR CONTROL - PROBLEM SET 2

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Problem 1. *Compute the Lie brackets for the following pairs of vector fields:*

(1) $v_1 = \begin{bmatrix} x_2 \\ x_1 + x_2 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} x_3 \\ x_1 \\ x_1 \end{bmatrix}$

(2) $v_1 = \begin{bmatrix} x_2 \\ x_1 + x_2 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Problem 2. *Consider the nonlinear system:*

$$\dot{x}_1 = x_1^2 x_2$$
$$\dot{x}_2 = 3x_2 + u$$

(1) *Taylor linearize the system at the origin, $u = 0, x_1 = 0, x_2 = 0$. Is the linearized system controllable?*
(2) *Is the nonlinear system exactly feedback linearizable? If so find the normal form and linearizing control.*
(3) *Consider the output $y = -2x_1 - x_2$. Find the input-output linearizing control. Obtain the zero dynamics and examine their stability. Can the system be asymptotically stabilized?*

Problem 3. *Consider a SISO system*

$$\dot{x} = f(x) + g(x)u, \quad y = h(x)$$

Assume the system has well-defined relative degree r, so that the feedback linearized input-output dynamics are:

$$y^{(r)} = v$$

Suppose $y_R(t)$ is a smooth (continuous derivatives up to $y_R^{(r)}$ will suffice) reference trajectory. Design a tracking controller based on stabilizing the error dynamics. Discuss your analysis.