Quadcopter Control Design

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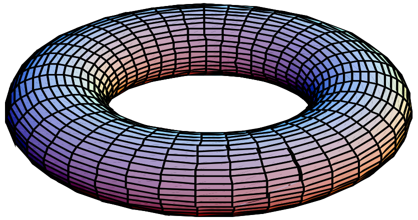

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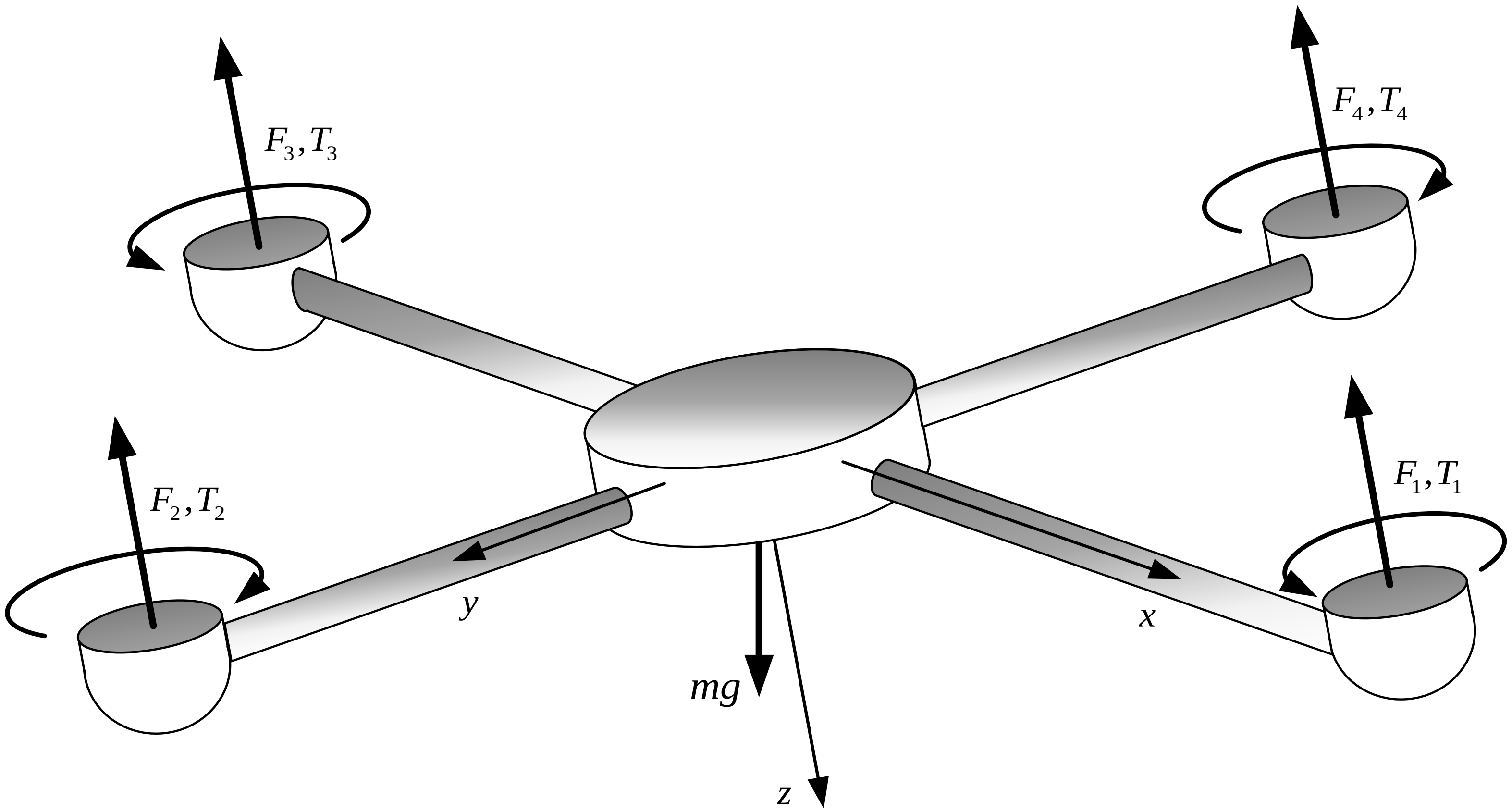
Outline

- The basic quadcopter
- The math model
- Generic control structure
- Project Objectives

Quadcopter Configuration



Kinematics

rotational

$$\frac{d}{dt}\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}=\begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}\quad\begin{matrix}\text{roll}\\ \text{pitch}\\ \text{yaw}\end{matrix}$$

translational

$$\frac{d}{dt}\begin{bmatrix}x_s\\ y_s\\ z_s\end{bmatrix}=\left[\begin{array}{c|c|c}\cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi-\cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi\\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi+\cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi\\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta\end{array}\right]\begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

Dynamics

rotational

$$\frac{d}{dt}\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \frac{I_y - I_z}{I_x} qr \\ \frac{I_z - I_x}{I_y} pr \\ \frac{I_x - I_y}{I_z} pq \end{bmatrix} + \begin{bmatrix} \frac{\ell}{I_x} u_2 \\ \frac{\ell}{I_y} u_3 \\ \frac{1}{I_z} u_4 \end{bmatrix}$$

translational

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} rv - qw - g \sin\theta \\ pw - ru + g\cos\theta \sin\phi \\ qu - pv + g\cos\phi\cos\theta \end{bmatrix} + \frac{1}{m}\begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix}$$

Controls

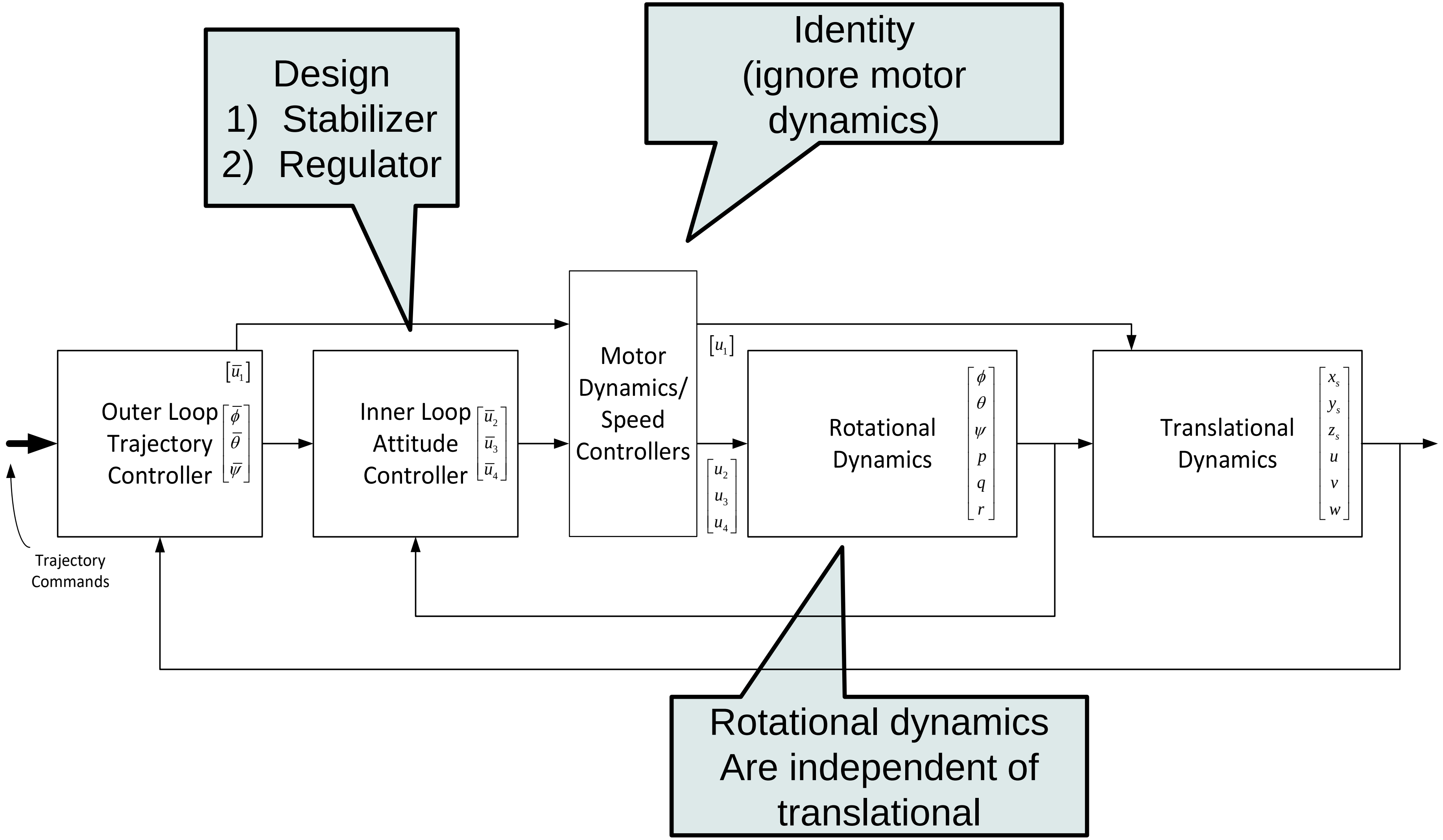
$$\text{roll:} \quad u_1 = \ell\left(F_4 - F_2\right) = \ell k\left(\omega_4^2 - \omega_2^2\right)$$

$$\text{pitch:} \quad u_2 = \ell\left(F_3 - F_1\right) = \ell k\left(\omega_3^2 - \omega_1^2\right)$$

$$\text{yaw:} \quad u_3 = -T_1 + T_2 - T_3 + T_4 = \kappa\left(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2\right)$$

$$\text{lift:} \quad u_4 = F_1 + F_2 + F_3 + F_4 = k\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right)$$

Control Configuration



Control Design

Rotational Stabilizer: Use Lyapunov method to determine a feedback control to stabilize the equilibrium point

$$\bar{p} = 0, \bar{q} = 0, \bar{r} = 0, \phi = 0, \theta = 0, \psi = \bar{\psi} = \text{const.}$$

Rotational Tracking: Design a regulator to track the commands

$$\bar{\psi}(t) = w_1(t), \bar{\theta}(t) = w_3(t), \bar{\phi}(t) = w_5(t)$$

$$\dot{w}_1 = w_2 \quad \dot{w}_3 = w_4$$

$$\dot{w}_2 = 0 \quad \dot{w}_4 = -w_3$$

$$\dot{w}_5 = 0$$

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