Nonlinear Modeling and Analysis Software for Control Upset Prevention and Recovery of Aircraft

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Modeling and simulation technologies perform an important role in understanding how aircraft depart from nominal flight and in validating and verifying the design of recovery techniques. Aircraft in-flight loss-of-control incidents often involve excursions into nonlinear flight regimes and so methods used to analyze them must include nonlinear tools. It has been previously shown that continuation methods can be employed very effectively to generate the equilibrium surface by varying parameters of interest. On this surface we can identify various point sets of interest, such as points where local stability characteristics are significantly altered, limits of actuation and control bifurcation points. We describe a continuation method that allows us to unravel the complicated equilibrium structure related to aircraft calculations. In this paper, we also describe software developed for the nonlinear analysis of impaired aircraft and control design for recovery methods. The software provides tools to assemble smooth full envelope, six degree-of-freedom nonlinear models based on multivariate orthogonal polynomial aerodynamics, to perform trim computations, to analyze singular points, to design nonlinear controllers, to assemble and execute simulations and to visualize results. The software can be used in mishap investigation, in assessing maneuverability envelopes and validating and verifying control techniques for recovery. It can also serve as a guide to aid experimentation by identifying the flight envelope of particular interest. We also present some calculations based on the Generic Transport Model (GTM) developed by NASA.

Nomenclature

$\mathbf{x}$	State vector
$\mathbf{y}$	Measurements ($= \mathbf{g}(\mathbf{x})$)
$\mathbf{z}$	Regulated variables ($= \mathbf{h}(\mathbf{x})$)
μ	Bifurcation parameter
$\mathbf{u}$	Control inputs
α	Angle of attack, rad
β	Side slip angle, rad

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V	Velocity, ft/s
p	X body-axis angular velocity component, rad
q	Y body-axis angular velocity component, rad
r	Z body-axis angular velocity component, rad
u	X body-axis translational velocity component, ft/s
v	Y body-axis translational velocity component, ft/s
w	Z body-axis translational velocity component, ft/s
ϕ	Euler roll angle, rad
θ	Euler pitch angle, rad
ψ	Euler yaw angle, rad
Ψ	Heading, rad
Γ	Flight path angle, rad
A	System matrix of a linear system
B	Control matrix of a linear system
C	Output matrix of a linear system
J	Jacobian matrix of a multivariable vector function
$\lambda, \tilde{\mathbf{v}}$	Eigenvalue and eigenvector of the Jacobian matrix J
T	Thrust, lb
T_c	Commanded Thrust, lb
δ_e	elevator input, rad
δ_a	Aileron input, rad
δ_r	Rudder input, rad

I. Introduction

Nonlinear modeling and analysis can provide great insight into off-nominal aircraft behavior. In-flight loss-of-control incidents often involve excursions into nonlinear regimes and the methods for mishap investigation, fault tolerant design and validation and verification must employ nonlinear techniques. It has been previously shown that continuation methods can be employed to generate the equilibrium surface by varying parameters of interest.[1] On this surface we can mark various regions of interest such as points where local stability characteristics are significantly altered, actuator limits and control related bifurcations points. These control bifurcation points are usually accompanied by an increased difficulty in regulating the aircraft and are related to the *zero structure* of the system.[1,2] It has also been shown that the problem of flight control reconfiguration following actuator failure can been formulated as a nonlinear regulator problem.[3,4] The post-fault controller uses the remaining functional actuators. It is designed to regulate key flight parameters while rejecting the disturbance induced by the failed actuator. The idea is that the pilot would maneuver the impaired aircraft by specifying the desired flight parameters. The post-fault system dynamics however can differ significantly from normal conditions.

In order to evaluate the ability of the impaired aircraft to maneuver we have previously employed continuation methods using a combination of Newton-Raphson (NR) and Newton-Raphson-Seydel (NRS)[5] methods. This method by varying a single parameter such as airspeed, center of mass, or flight path angle generates a codimension one surface in the space of states and (functional) controls.[1] However, even these surfaces can have complicated folds and turns that cannot be easily accommodated by NR/NRS techniques. We have developed a novel continuation method using a technique previously used to generate linear parameter varying families from parameter dependent nonlinear dynamics.[6] The essential idea is to define a local coordinate system on the equilibrium surface and utilize the new parametric representation to unravel the structure of the surface in the neighborhood of multiple folds where earlier methods failed. However, it is important to note that the region of interest in these surfaces tends to be the first control bifurcation and the various actuator limits that are ordinarily found before the more complex surface structures are encountered. These limits define the maneuverability envelope associated with the aircraft. The static bifurcation points that occur in a control system are associated with the *zero structure* of the system. The approach to bifurcation is accompanied by an increasing difficulty to regulate the aircraft. We can locate the bifurcation points accurately in order to assess the specific degeneracy.

We apply the method to the study of a six degree of freedom Generic Transport Model (GTM). The symbolic modeling of the GTM is facilitated by the polynomial aerodynamic models developed in 7. We have developed the model in *Mathematica*®.[8] The symbolic model is used to generate the equilibrium curves. In the analysis presented here we consider straight and level flight with velocity as the parameter. Other motions that have been studied are wings level flight climbing and descending, coordinated turns and steady sideslip in climb and descent.

To support this effort we have developed a comprehensive software package for the nonlinear analysis of normal and impaired aircraft. The software provides tools to assemble smooth full envelope, six degree-of-freedom nonlinear models based on multivariate orthogonal polynomial aerodynamics, to perform trim computations, to analyze singular points, to design nonlinear controllers, to assemble and execute simulations and to visualize results. The software can be used as the first step in validating and verifying maneuverability envelopes and control techniques for recovery. It can also serve as a guide to aid experimentation by identifying the interesting parts of the flight envelope. This software leverages the strengths of widely available scientific software such as *Mathematica* and *MATLAB/Simulink*®.[9] The associated graphical user interface and database tools for models are developed using cross platform, plug-in architecture. The computational tools for model assembly and analysis are provided in symbolic environment which also has tools for nonlinear control design. Closed loop simulation models can be generated from the front-end and transferred to *MATLAB/Simulink*. In addition to this, 3D tools are provided to aid visualization of the aircraft behavior.

The rest of the paper is organized as follows. Bifurcation analysis in the context of control systems, and the continuation method to carry out the analysis is discussed in Section II. The nonlinear six degree of freedom GTM model and the analysis presented in Section III. Section IV details the some of the features of the developed software. Section V is the conclusion.

II. Bifurcation analysis of control systems

In this paper we consider the regulation of aircraft to a desired trim condition. In general the equations of motion of a rigid aircraft involve six degrees of freedom involving the six coordinates $\phi, \theta, \psi, x, y, z$ and six (quasi-) velocities p, q, r, u, v, w. In the study of steady motions it is usual to ignore the location (x, y, z) and consider only the velocities and attitude of the vehicle. The reduced dynamics comprise a nine dimensional system of state equation that are nonlinear and may be parameter dependent. We are concerned with steady motions that can be defined in terms of these variables. In particular, these motions are trajectories in the x, y, z space, that can be associated with equilibria of the nine state system. We would like to know whether or not it is possible to regulate to and steer along these motions. Thus, we are naturally lead to the study of the existence and stabilizability of equilibria.

A. Bifurcation of equilibria in control systems

Consider a parameter dependent, nonlinear control system given by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mu) \\ \mathbf{z} &= \mathbf{h}(\mathbf{x}, \mu) \end{aligned} \qquad (1)$$

where $\mathbf{x} \in R^n$ are the states, $\mathbf{u} \in R^p$ are the control inputs, $\mathbf{z} \in R^r$ are the regulated variables and $\mu \in R$ is any parameter. We assume that $\mathbf{f}$, $\mathbf{h}$ are smooth (sufficiently differentiable). The parameter could be a physical variable like the weight of the aircraft or the center of gravity location; or a regulated variable like velocity, flight path angle, altitude or roll angle; or a stuck control surface. The regulator problem is solvable only if $p \geq r$. Since the number of controls can always be reduced we henceforth assume $p = r$.

A triple $(\mathbf{x}^\star, \mathbf{u}^\star, \mu^\star)$ is an *equilibrium* point of (1) if

$$F(\mathbf{x}^\star, \mathbf{u}^\star, \mu^\star) := \begin{pmatrix} \mathbf{f}(\mathbf{x}^\star, \mathbf{u}^\star, \mu^\star) \\ \mathbf{h}(\mathbf{x}^\star, \mu^\star) \end{pmatrix} = \mathbf{0} \tag{2}$$

Definition 2.1 [10] *An equilibrium point $(\boldsymbol{x}^\star, \boldsymbol{u}^\star, \mu^\star)$ is regular if there is a neighborhood of $\mu^\star$ on which there exist unique, continuously differentiable functions $\overline{\boldsymbol{x}}(\mu)$, $\overline{\boldsymbol{u}}(\mu)$ satisfying*

$$F(\overline{\boldsymbol{x}}(\mu), \overline{\boldsymbol{u}}(\mu), \mu) = \boldsymbol{0} \tag{3}$$

If an equilibrium point is not a regular point it is a *static bifurcation point.* The Implicit Function Theorem implies that an equilibrium point is a bifurcation point only if $\det J = 0$. The Jacobian J is given by

$$J = [D_{\mathbf{x}} F(\mathbf{x}^\star, \mathbf{u}^\star, \mu^\star) \quad D_{\mathbf{u}} F(\mathbf{x}^\star, \mathbf{u}^\star, \mu^\star)] \tag{4}$$

Now, if A, B, C, D denotes the linearization at $(\mathbf{x}^\star, \mathbf{u}^\star, \mu^\star)$ of (1) with output $\mathbf{z}$ so that

$$J = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$$

Then we have the following theorem for a static bifurcation point.

Theorem 2.2 [10] *An equilibrium point $(\boldsymbol{x}^\star, \boldsymbol{u}^\star, \mu^\star)$ is a static bifurcation point only if*

$$Im \begin{pmatrix} A & B \\ C & 0 \end{pmatrix} \neq R^{n+r} \tag{5}$$

Recall that the system matrix is

$$P(\lambda) = \begin{pmatrix} \lambda I - A & B \\ -C & 0 \end{pmatrix}$$

From this observation, *necessary* conditions for a static bifurcation point can be obtained as follows:[2]

Theorem 2.3 *The equilibrium point $(\boldsymbol{x}^\star, \boldsymbol{u}^\star, \mu^\star)$ is a static bifurcation point of (1) only if one of the following conditions is true for its linearization:*

1. *there is a transmission zero at the origin,*
2. *there is an uncontrollable mode with zero eigenvalue,*
3. *there is an unobservable mode with zero eigenvalue,*
4. *it has insufficient independent controls,*
5. *it has redundant regulated variables.*

It turns out that static bifurcations in regulators are always associated with a degeneracy in the linearized system zero dynamics. Such degeneracies include the loss of linear observability or controllability. But this does not imply that the system fails to be observable or controllable in a nonlinear sense. However, if the bifurcation is associated with a breakdown in linear controllability or observability means that for a linear controller change of input-output structure for the regulator problem is needed for design. Even if system continues to be controllable and observable in the nonlinear sense it precludes the possibility of smooth control action.[2] This connection with the input-output structure is the primary difference when considering bifurcations in control systems rather than dynamical systems. For discerning the maneuverability of the system we are interested in limits arising not just from static bifurcation points but also loss of stability and actuation limits. The equilibrium equations are simpler for the open loop system. Once we have obtained the bifurcation curves for the open loop system, the closed loop bifurcation curves can be obtained using the control law.

B. Computing bifurcation diagrams

Consider the system of algebraic equations:

$$F(x, \mu) = 0 \tag{6}$$

where $F : R^{n+1} \to R^n$, $x \in R^n$, and $\mu \in R$. We wish to generate a graph of x versus μ. In general, one can start with a specific parameter value, μ_0, and compute the corresponding $x_0 = x(\mu_0)$ that satisfies $F(x_0, \mu_0) = 0$. This could be done by applying the Newton-Raphson (NR) method with an initial estimate, x_0^0. Then, the obvious procedure is to increment the parameter $\mu_0 \to \mu_1 = \mu_0 + \Delta\mu$ and again apply the NR method with initial estimate $x_1^0 = x(\mu_0)$ to obtain $x_1 = x(\mu_1)$ that satisfies $F(x_1, \mu_1) = 0$. One continues in this way to generate the sequence of pairs $\{(\mu_0, x_0), (\mu_1, x_1), (\mu_2, x_2), \ldots\}$, thereby generating the graph data.

Unfortunately the procedure is almost certain to fail. A generic, smooth function (6) will contain fold bifurcations. Thus, no matter how small the increment $\Delta\mu$, sooner or later there will be no values x_{i+1} near $x_i$that satisfies $F(x_{i+1}, \mu_i + \Delta\mu) = 0$. The situation is illustrated in Figure .1. At the bifurcation point the Jacobian of F with respect to x is singular, so even with sufficiently small steps as the bifurcation point is approached the NR iteration breaks down. One remedy for this is the Newton-Raphson-Seydel (NRS) method[5] which has proved useful in some applications.[11] The strategy, is to switch to the NRS method near the bifurcation point and track the smallest eigenvalue as the curve passes through the bifurcation point, and the eignenvalue through zero.

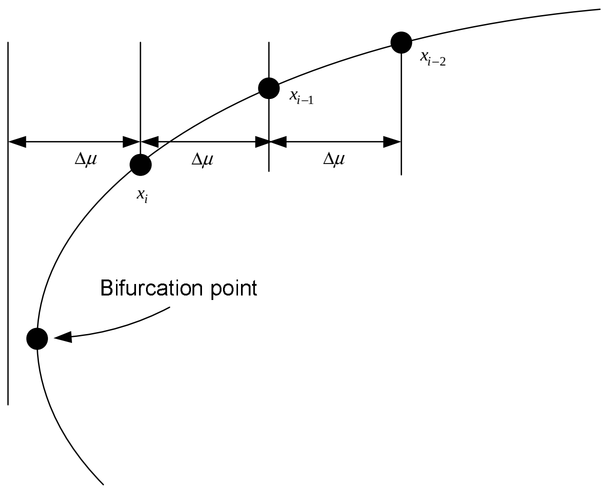

Figure .1. Computing near a fold bifurcation.

The above approach assumes that the smallest eigenvalue of the Jacobian, evaluated at the point where NR breaks down, is the one that eventually goes to zero at the bifurcation point. When two or more eigenvalues are close to the origin it is not clear which one will reach the origin eventually. Also, if the eigenvalues closest to the origin are a complex pair, the path they take to the real axis, so that one of them can eventually wind up at the origin, is not known a priori. Sufficiently close to the bifurcation point this ambiguity gets resolved and typically only one eigenvalue approaches zero and then the associated eigenvector can be used to proceed with the analysis using the NRS method. But ‘sufficiently close’ is not well-defined for computational purposes.

It should also be noted that an eigenvalue of the Jacobian may become small because it is close to another branch of the bifurcation diagram. In such instances, allowing the continuation parameter to approach zero results in the computed equilibrium to lie on the other branch of the curve. Also, in cases where there are two consecutive bifurcation points on the bifurcation curve, the smallest eigenvalue may move away from the origin and approach the origin from the same side, as we progress from the first to the second bifurcation point. An interesting complexity is encountered when two eigenvalues approach zero along,or very near, the imaginary axis. We have observed all of these situations in studying aircraft bifurcations. Consequently, we use an alternative to the eigenvalue tracking strategy. Our method is described below.

First, initialize as follows. Specify the parameter range by defining μ_{min} and $\mu + max$. Define maximum (starting) and minimum parameter increments,$\Delta\mu_{\min}$, and $\Delta\mu_{\min}$. Specify a starting point x_0, μ_0 that satisfies (6), and a starting direction, d, for the parameter (d $= \pm 1$). Then proceed as follows.

1. Given (μ_i, x_i):
   (a) If $\mu_i \notin [\mu_{\min}, \mu_{\max}]$, Stop.
   (b) Else, at point (μ_i, x_i)obtain a local parametric approximation to the equilibrium surface; $s \in R \to \mu(s), x(s)$. We use the method of[6] to construct a cubic approximation.
2. Solve $\mu(s) = \mu_i + \Delta\mu$.
   (a) If there are real solutions of the same sign as previous step, take the smallest, $s_{\min}$, set $\mu_{i+1} = \mu(s_{\min})$, and apply NR to $f(x_{i+1}, \mu_{i+1}) = 0$ to obtain x_{i+1}. Return to Step 1.
   (b) Else, there are no real solutions of proper sign, replace $\Delta\mu \to \Delta\mu/2$ and try (a) again, until $\Delta\mu < \Delta\mu_{\min}$. Then go to Step 3.
3. Solve the equation $d\mu(s)/ds = 0$ to obtain real $s = s^*$ of the same sign as previous step. See Remark 2.4
   (a) If, $|\mu(s^*) - \mu_i| < \Delta\mu_{\min}$, go to Step 4.
   (b) Else, solve $\mu(s) = \mu_i + \mathrm{d}\Delta\mu_{\min}$ using NR to obtain real $s = \widehat{s}$ of same sign as previous step. Determine the eigenvalue of least magnitude, λ_0, of Jacobian $\partial F\left(x\left(\widehat{s}\right), \mu(\widehat{s})\right)/\partial x$, and its associated eigenvector, $\mathbf{v}_0$. Set up Newton-Raphson-Seydel (NRS) equations in the form:

$$\begin{array}{l} F(x,\mu) = 0 \\ \frac{\partial F(x,\mu)}{\partial x}\mathbf{v} = \lambda_0 \mathbf{v} \\ \|\mathbf{v}\| = 1 \end{array} \tag{7}$$

   Solve for $x, \mu, \mathbf{v}$using NR with initial estimate $x_i, \mu_i, \mathbf{v}_0$. Set $x_{i+1} \to x, \mu_{i+1} \to \mu$, update the function $\mu(s)$ and repeat 3.
4. Precisely locate bifurcation point by solving (7) with $\lambda_0 = 0$ and using last computed $x, \mu, \mathbf{v}$. Save result and proceed to 5.
5. Reverse direction, d $\to -$d, and solve $\mu(s) = \mu_i + \mathrm{d}\Delta\mu_{\min}$ using NR to obtain real $s = \widehat{s}$ of same sign as previous step. Note that $\mu(s)$ was determined before the bifurcation point. Determine the eigenvalue of least magnitude,λ_0, of Jacobian $\partial F\left(x\left(\widehat{s}\right), \mu(\widehat{s})\right)/\partial x$, and its associated eigenvector, $\mathbf{v}_0$. Solve (7) as in Step 3. Return to Step 1.

Remark 2.4 *Note that since $\mu(s)$ is a cubic with real coefficients it can be proved that for any μ_i and any increment $\Delta\mu$ in the direction of progress, there is either real solution of $\mu(s) = \mu_i + \Delta\mu$ in the direction of progress or an extreme point, a solution of $d\mu(s)/ds = 0$, in the direction of progress.*

III. Six degree of freedom Generic Transport Model and analysis

The aircraft model has 9 states, given by $\mathbf{x} = [\phi\ \theta\ \psi\ p\ q\ r\ u\ v\ w]^T$, and 4 control inputs, given by $\mathbf{u} = [T, \delta_e, \delta_a, \delta_r]^T$. The controls are limited as follows: thrust $0 \leq T \leq 35.4372\ lbf$, elevator $-40° \leq \delta_e \leq 20°$, aileron $-20° \leq \delta_e \leq 20°$, and rudder $-30° \leq \delta_e \leq 30°$. Velocity is in ft/s

The state space representation of the model is

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$$

Aerodynamic models were based on wind tunnel data obtained with a 5.5 percent scale model in the NASA Langley 14 ft X 22 ft tunnel. Aerodynamic force and moment coefficients are based on a polynomial formulation using multivariate orthogonal function method as decribed in 12 and 13. In the original model several breakpoints in angle of attack were used to capture the nonlinearity and to make sure the endpoints of each subspace were modeled with an approximately correct sign from each side of the subspace boundary. The subspace models were blended using Gaussian weighting, as was done for the static data. For simplicity we use only one of the models for analysis. The characteristics of the full scale airplane are achieved by specific dynamic scaling requirements imposed on the subscale aircraft.[14] The trim computations and bifurcation analysis are performed on the open loop model. Trim computations are performed for level flight. This computed trim is used as the initial starting point for the algorithm described in the previous section. For the curves presented below it is $(\phi, 0.), (\theta, 0.149134), (\psi, 0.), (p, 0.), (q, 0.), (r, 0.), (V, 100.), (\alpha, 0.149134), (\beta, 0.)$ and $(T, 3.95663), (\delta_e, -0.0951322), (\delta_a, 0.), (\delta_r, 0.)$.

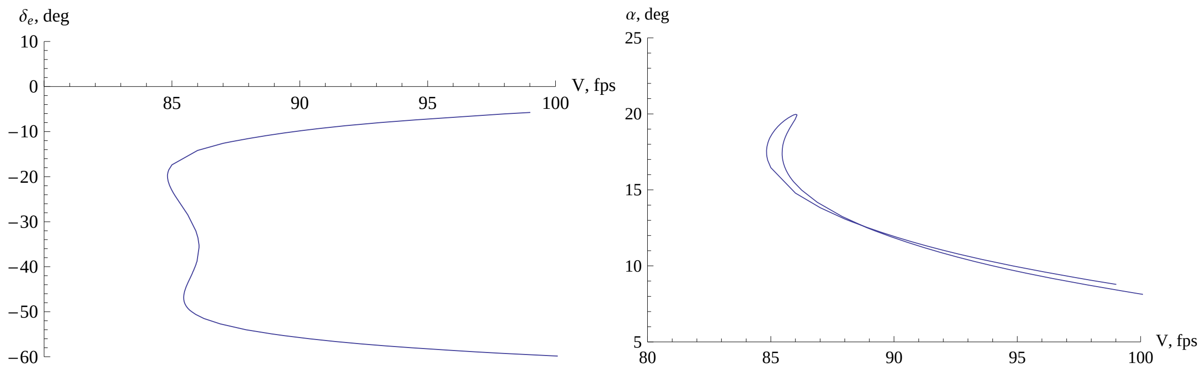

Figure .2. Bifurcation curves for elevator and angle of attack Generic Transport Model in level flight with velocity as a parameter

As shown in Figures .2 and .3, as velocity is decreased from a nominal value the equilibrium curve indicates several bifurcation best discerned when the elevator deflection is plotted versus velocity. As the velocity is decreased from 100 fps along the upper branch – which represent the normal trim conditions – it approaches a bifurcation point that characterizes a classical stall condition. As stall is approached the aircraft pitches up, and angle of attack increases as does thrust. The increase in thrust with decrease in velocity is not unexpected at very low speed. In this case the limits for the actuators are not reached before the model reaches stall.

The equilibrium surface is a co-dimension one surface in the space of dependent variables. The number of dependent variables can vary depending on the model and the specific scenario. In the case just described there are 4. In general, the structure of the surface can be examined by plotting each of the variables against the parameter, in this case, velocity. The limits of actuation can also be marked on these curves in order to isolate the relevant equilibria. The stability of the equilibria can be ascertained through examination of the

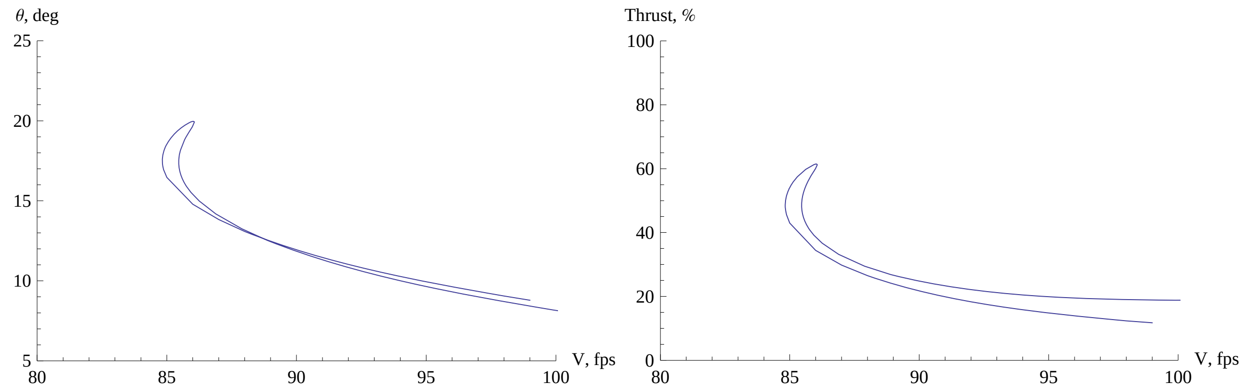

Figure .3. Bifurcation curves for pitch angle and thrust Generic Transport Model in level flight with velocity as a parameter

eigenvalues of the linearized system. Unlike dynamical systems, bifurcation of equilibria in control systems does not correspond to a change in eigenvalue structure.

IV. Software design

We have designed software with a cross platform front end to guide users define an aircraft flight dynamics problem or to utilize stored model definitions from a database. The software is developed using rich client platform tools and architecture is based on *plug-ins*. We have developed a wizard for model entry and manipulation as well as visualization tools for plotting the curves. *Plug-ins* also provide connectivity to tools developed in *Mathematica* and *MATLAB/Simulink*. The software is intended for use by engineers and scientists, who need to be mobile, work offline, integrate content, collaborate and take advantage of local or other available computing resources. The database tools associated with the software can store both the mathematical models and generated results to aid flexibility.

For new model definitions a wizard guides the user through various necessary steps in defining the model. The model definition begins by storing some general informaion about the model. The model is defined by entering the various inertial parameters. The multivariate polynomials that define the aerodynamic behavior of the aircraft can be entered or can be generated from data. When generating the polynomials from data, selected orthogonal modeling functions are included in the model based on minimizing the predicted square error. The final identified model consists of selected terms from the multivariate power series expansion for the dependent variable in terms of the independent variables. The software provides flexibility in entering the functional descriptions by creating all the terms of the polynomial series automatically. The user then selects the appropriate terms for the current model. The user has the flexibility to add or remove control input variables in the framework. The default configuration is customizable making the software flexible in the ability to accomodate wide range of aircraft models.

After the model parameters and aerodynamic forces and moments are entered the user can assemble the model using a connection to *Mathematica* kernel. *Mathematica* functions have been created to assemble the smooth model. Additionally, functions to enable trim computations, generation of linear parameter varying models are also provided. Using assumption of flat earth and rigid body for the aircraft under which the smooth aerodynamic model applies, we can assemble the nine fundamental equations of motion that include the force, moment balance and the three kinematic equations for the propagation of the Euler angles, which describe the attitude of the aircraft, and the rotations are measured from a right handed inertial frame whose

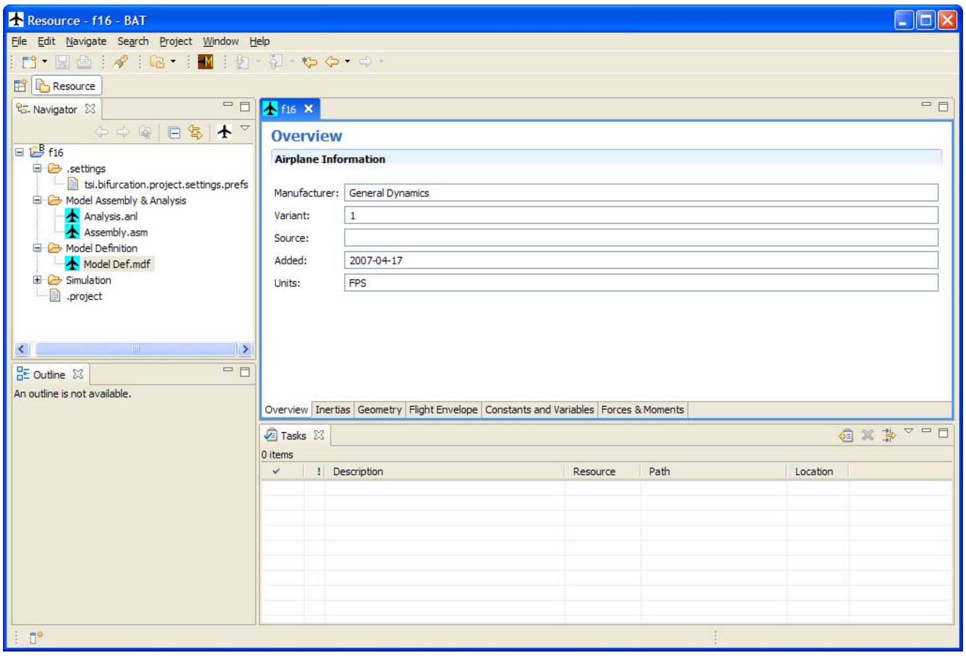

Figure .4. Model definition using the Java front end of the designed software.

z-axis points to the center of the earth.

The tools for nonlinear analysis are designed to identify control bifurcation points as they relate to the control of regulated variables, stability limits in the dynamics and saturation of the control surfaces. The continuation parameters that can be used are set points of regulated variables (such as velocity, flight path angle, roll angle, etc.), physical aircraft parameters (such as total weight, center of mass, relative location of center of gravity and aerodynamic center, etc.) or for mishap investigation or safety analysis change in the actuation parameters (stuck position of the actuation surfaces, etc.). State-space models can be obtained by inverting the non-singular mass matrix. The bifuration curves can be obtained by previously described global reparameterization of the equilibrium surface. The data from analysis can be stored in a database and can be plotted using an Open GL library which is a part of the software. Finally, the models can be transferred to *MATLAB/Simulink* for simulation and a 3D viewer for visualization.

V. Conclusions

We have developed a novel continuation procedure to unravel the control bifurcation structure for aircraft models. Nonlinear analysis techniques have been implemented in symbolic software to aid aircraft safety goals of control upset prevention and recovery of aircraft. The computational tools can also be employed in mishap investigation and to guide flight experiments. An example calculation using these tools is presented for the Generic Transport Model. These tools have been integrated into a stability and bifurcation analysis software which includes a front end and visualization tools.

References

[1]Thomas, S., Bajpai, G., H., K., and C., C. B., "Nonlinear Dynamics, Stability and Bifircation in Aircraft: Simulation and Analysis Tools," *Guidance, Navigation and Control*, No. 6428, AIAA, San Francisco, CA, August 2005.

[2]Kwatny, H. G., Chang, B. C., and Wang, S. P., "Static Bifurcation in Mechanical Control Systems," *Chaos and Bifurcation Control: Theory and Applications*, edited by G. Chen, Springer-Verlag, 2003.

[3]Bajpai, G., Chang, B. C., and Kwatny, H. G., "Design of fault-tolerant systems for actuator failures in nonlinear systems," *American Control Conference*, Vol. 5, IEEE, May 2002, pp. 3618–3623.

[4]Thomas, S., Kwatny, H. G., and Chang, B. C., "Nonlinear Reconfiguration for Asymmetric Failures in a Six Degree-of-Freedom F-16," *American Control Conference*, IEEE, Boston, MA, June/July 2004, pp. 1823–1829.

[5]Seydel, R., *Practical Bifurcation and Stability Analysis*, Springer Verlag, New York, 1994.

[6]Kwatny, H. G. and Chang, B. C., "Constructing Linear Families from Parameter-Dependent Nonlinear Dynamics," *IEEE Transactions on Automatic Control*, Vol. 43, No. 8, 1998, pp. 1143–1147.

[7]Morelli, E. A., "Generic Transport Model Aircraft Nonlinear MATLAB Simulation based on Polynomial Aerodynamic and Engine Models," Readme file, gtm polysim 1.0, NASA Langley Research Center, January 2007.

[8]"Wolfram Research," [http://www.wolfram.com/].

[9]"The Mathworks," [http://www.mathworks.com/].

[10]Kwatny, H. G., Bennett, W. H., and Berg, J. M., "Regulation of Relaxed Stability Aircraft," *IEEE Transactions on Automatic Control*, Vol. AC-36, No. 11, 1991, pp. 1325–1323.

[11]Kwatny, H. G., Fischl, R. F., and Nwankpa, C., "Local Bifurcations in Power Systems: Theory, Computation and Application," *Proceedings of the IEEE*, Vol. 83, No. 11, November 1995, pp. 1456–1483.

[12]Morelli, E. A., "Global Nonlinear Aerodynamic Modeling Using Multivariate Orthogonal Functions," *Journal of Aircraft*, Vol. 32, No. 2, 1995, pp. 270–277.

[13]Morelli, E. A. and DeLoach, R., "Wind Tunnel Database Development using Modern Experiment Design and Multivariate Orthogonal Functions," *41^{s}t AIAA Aerospace Sciences Meeting and Exhibit*, AIAA Paper 2003-0653, Reno, NV, January 2003.

[14]Jordan, T., Langford, W., Belcastro, C., Foster, J., Shah, G., Howland, G., and Kidd, R., "Development of a Dynamically Scaled Generic Transport Model Testbed for Flight Research Experiments," *AUVSI Unmanned Unlimited*, Arlington, VA, 2004.