Voltage Stability Toolbox for Power System Education and Research

Saffet Ayasun, *Member, IEEE*, Chika O. Nwankpa, *Senior Member, IEEE*, and Harry G. Kwatny, *Life Fellow, IEEE*

***Abstract*—This paper presents a Matlab-based voltage stability toolbox (VST) designed to analyze bifurcation and voltage stability problems in electric power systems. VST combines proven computational and analytical capabilities of bifurcation theory, and symbolic implementation and graphical representation capabilities of Matlab and its toolboxes. The motivation for developing the package is to provide a flexible simutlation environment for an ongoing research conducted at the Center for Electric Power Engineering (CEPE) of Drexel University, Philadelphia, PA, and to enhance undergraduate/graduate power engineering courses. VST is a very flexible tool for load flow, small-signal and transient stability, and bifurcation analysis. After a brief summary of power system model and local bifurcations, the paper illustrates the capabilities of VST using the IEEE 14-bus system as an example and describes its successful integration into power engineering courses at Nigde University, Nigde, Turkey.**

***Index Terms*—Load flow, local bifurcations, Matlab, power engineering education, symbolic computation, voltage stability.**

I. Introduction

The methods of bifurcation theory can be effectively used to analyze various types of stability problems in power systems, such as voltage stability and collapse and oscillatory phenomena [1]–[8]. Therefore, bifurcation analysis has become an important tool in practical analysis of power system stability [9]–[13].

Power system stability analysis using bifurcation theory is usually taught in the graduate level courses. Teaching bifurcation theory and its applications to stability analysis is a major undertaking that involves a multidisciplinary approach in which the fundamentals of power system modeling together with diverse mathematical formulations of bifurcation theory need to be merged in a common framework. One convenient and yet powerful way is to use simulation tools as the common framework and to integrate them into power engineering courses [14]–[16]. Such a simulation tool needs to be user friendly and should make the powerful but complex bifurcation theory and its recent applications readily accessible to the students.

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Simulation tools for power system stability analysis can be divided into two classes—commercial programs and customized toolboxes developed for education and research. Various commercial programs, such as Power System Simulator for Engineering (PSS/E), Power System Simulator (Simpow), and PowerWorld, are available on the market. These programs provide detailed component/system models and computationally efficient algorithms for the analysis. However, they are not suitable for educational and research purposes since they usually do not allow modification or addition of new component models and algorithms. For education and research purposes, flexibility and ability of easy prototyping are often more crucial aspects than computational efficiency.

In the area of power systems, a Matlab [17] software package has become one of the most popular scientific programming languages for research and teaching applications. The following features make Matlab an attractive choice for power systems: 1) wide availability, portability, and cost; 2) high-quality numerical processes, including sparse matrix capability; 3) the ability to create dynamically linked C/C++ or FORTRAN subroutines; 4) the availability of symbolic computation capability through the Extended Symbolic Toolbox; 5) the ability to build a portable and powerful graphical user interface; 6) it is widely used in engineering curricula and well known by students; and 7) wide selection of toolboxes, such as Simulink, SimPowerSystems, Control Toolbox, and Real-Time Workshop. Several Matlab-based programs are available in power system simulation, modeling, and analysis, such as Power System Toolbox (PST) [18], Electromagnetic Transients Program in Matlab (MatEMTP) [19], Power Analysis Toolbox (PAT) [20], Educational Simulation Tool (EST) [21], SimPowerSystems (SPS) [22], and Matlab Power System Simulation Package (MatPower) [23]. Among these, only MatPower program is open source and freely downloadable.

This paper describes a new Matlab-based voltage stability toolbox (VST) that uses visualization capabilities of Matlab and integrates the symbolic and numeric computations to investigate voltage stability and bifurcation issues in power systems. VST is an open source simulation tool and is freely available at http://power.ece.drexel.edu, the website of the Center for Electric Power Engineering (CEPE) of Drexel University, Philadelphia, PA. VST was first designed to have a computational tool that supports the ongoing research on the voltage stability analysis at CEPE [24]–[26] and was improved progressively within the context of the Ph.D. dissertation [27], [28].

Table I gives a comparison of the currently available Matlab-based tools for power system analysis and VST. The features illustrated in the table are: load flow (LF), voltage stability analysis (VSA), small-signal stability analysis (SSA),

TABLE I
MATLAB-BASED SIMULATION TOOLS FOR POWER SYSTEM ANALYSIS

Tool	LF	VSA	SSA	TD	EMT	GUI
VST	√	√	√	√		√
PST	√		√	√		
MatEMTP				√	√	√
PAT	√		√	√		
EST	√		√	√		
SPS	√		√	√	√	√
MatPower	√					

time-domain (TD) simulation, electromagnetic transients (EMT), and graphical-user interface (GUI). As the table clearly indicates, the VSA function included in VST is a novelty among available Matlab-based packages for power system analysis. The VSA function, as will be described later in the paper, implements a continuation load flow algorithm that computes a set of equilibria (operating points), a nose curve for a given load, and/or a generation increase scenario and associated local bifurcations by monitoring the system eigenvalues. Moreover, convenient and powerful GUI of VST provides an easy-to-use simulation environment within which even a user not well versed in the mathematics of bifurcation theory or experienced in voltage stability analysis can easily experiment with standard systems or construct one of his or her own. An experienced user can exploit open architecture of VST to implement and experiment with alternative computational algorithms.

VST could be effectively used to enhance power engineering undergraduate/graduate courses. For undergraduate courses, the load flow module could be helpful to illustrate modeling (i.e., basic bus models and network admittance matrix) and algorithmic issues (Newton–Raphson (NR) method, convergence, multiple solutions, etc.) in load flow analysis. On the other hand, the bifurcation analysis module of VST could enhance graduate courses, especially courses that introduce students to subjects on active research areas, such as bifurcation theory and its application into voltage stability analysis. The enhancement can be achieved by using VST for various educational activities, such as classroom demonstration, exercises, and assignments. The classroom demonstration enables the instructor to illustrate to students the fundamental concepts of voltage stability phenomena, such as multiple equilibria, loss of equilibria, small-signal stability features of the equilibria, and local bifurcations. With exercises and assignments given, students will be able to investigate the effects of different loading conditions on the system stability and to learn how corrective control strategies could be developed to prevent instabilities. With the help of simulation results they obtain, students will increase their understanding of power system characteristics and behavior beyond the understanding they gain from classroom lectures and textbooks.

The remainder of the paper is organized as follows. Section II gives the differential-algebraic equation (DAE) model of power systems and summarizes local bifurcations of the equilibria. Section III describes the structure of VST and its applications using the IEEE 14-bus system. Section IV explains how VST was used in power engineering courses at Nigde University, Nigde, Turkey, and summarizes student evaluations, while the last section concludes the paper.

II. DIFFERENTIAL-ALGEBRAIC POWER SYSTEM MODEL AND LOCAL BIFURCATIONS

The DAE model is widely used to describe the dynamics of power systems for voltage stability analysis [5], [6]. For all analysis, VST implements the following DAE model of the classical power system with constant PQ load buses:

$$\begin{aligned} \dot{x} &= f(x, y, \beta) \\ 0 &= g(x, y, \beta) \end{aligned} \tag{1}$$

where x is the vector of state variables (generator angles and angular velocities), y is the vector of algebraic variables (voltage magnitude and phase angles at the load buses), and β is the vector of parameters (real/reactive power demand at the buses and transmission line parameters). The differential equation is the swing equation describing dynamics of each generator, and algebraic equations are the power flow equations representing real and reactive power balances at the load buses.

When parameters are subject to variations, the equilibria of the DAE power system model may exhibit three local bifurcations, namely saddle node (SN), Hopf and singularity induced (SI) bifurcations [5], [6]. The SN bifurcation occurs when a stable equilibrium point meets an unstable equilibrium point indicating the loss of equilibria. The SN bifurcation has become a widely accepted paradigm for one important form of voltage instability and linked to voltage collapse [29]. Hopf bifurcation occurs when the Jacobian matrix of the linearized model around an operating point has a pair of imaginary eigenvalues. Hopf bifurcation leads to oscillatory instabilities in power systems [30], [31]. The SI bifurcation occurs when system equilibria encounter the singularity manifold. The SI bifurcation refers to a stability change as the result of one eigenvalue of a reduced Jacobian matrix associated with the equilibrium diverging to infinity [6], [8], [11]. The infinite (unbounded) eigenvalue at this bifurcation implies that some of the slow dynamics of state variable (x) of the DAE model become very fast near the SI bifurcation. The presence of the eigenvalue in the right-half plane implies that the power system is small-signal unstable after the SI bifurcation.

III. VOLTAGE STABILITY TOOLBOX

VST was designed to analyze bifurcation and voltage stability problems in electric power systems. VST combines symbolic and numeric computations with a graphical menu-driven interface based on Matlab and its extended symbolic toolbox.

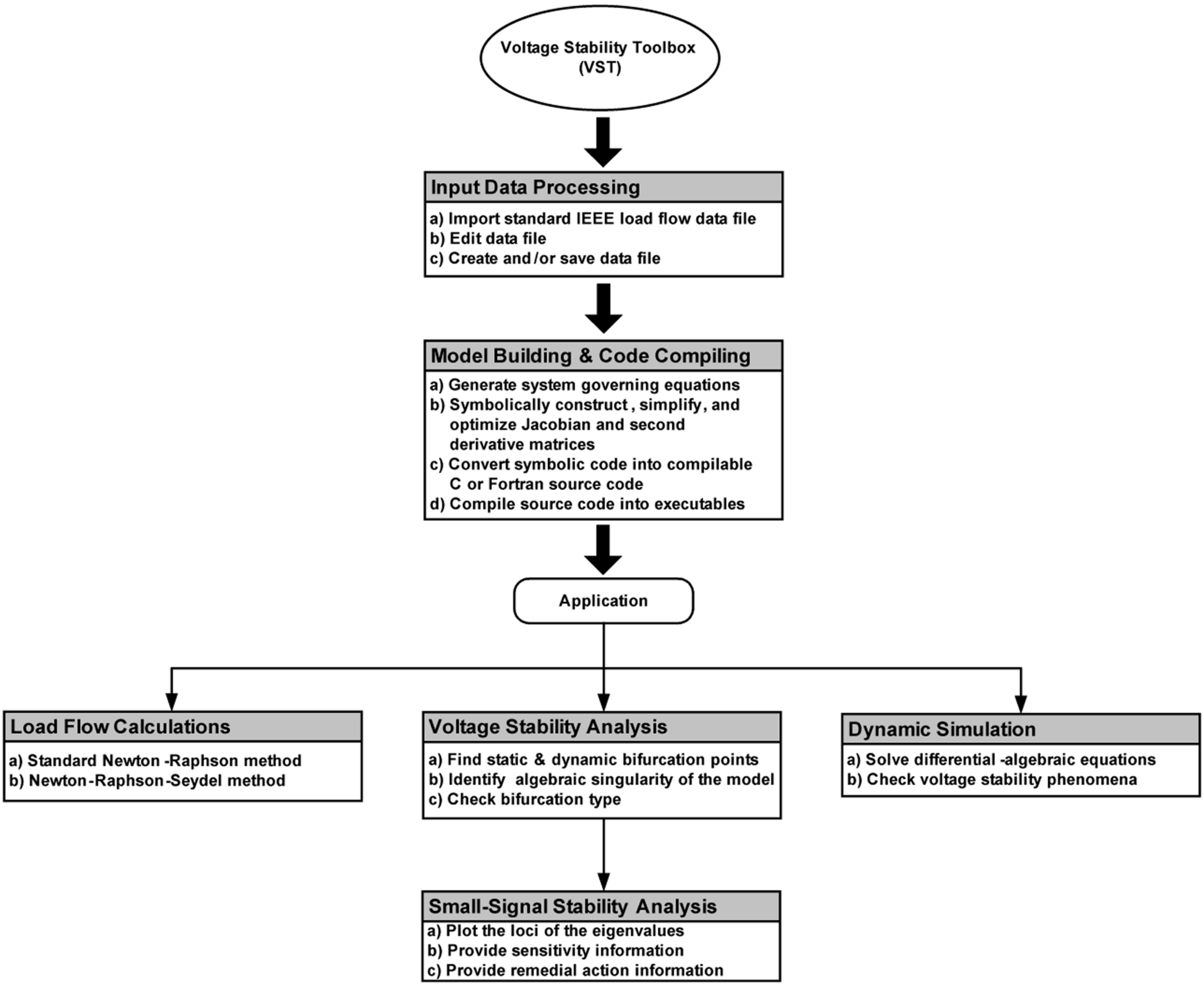


Fig. 1. VST software configuration.

The main features and application modules of VST can be summarized as follows.

- use of Matlab's visualization capability to create graphical user interface and visualize output data;
- use of stand-alone MEX-files to generate classical power system model equations;
- use of symbolic toolbox to generate Jacobian and second-order derivative matrices;
- load flow calculations: standard NR and convergent Newton–Raphson–Seydel (NRS) [32] methods;
- Voltage stability analysis: identification of local static and dynamic bifurcation points, such as SN, Hopf, and SI bifurcations;
- small-signal stability analysis;
- dynamic (time-domain) simulations.

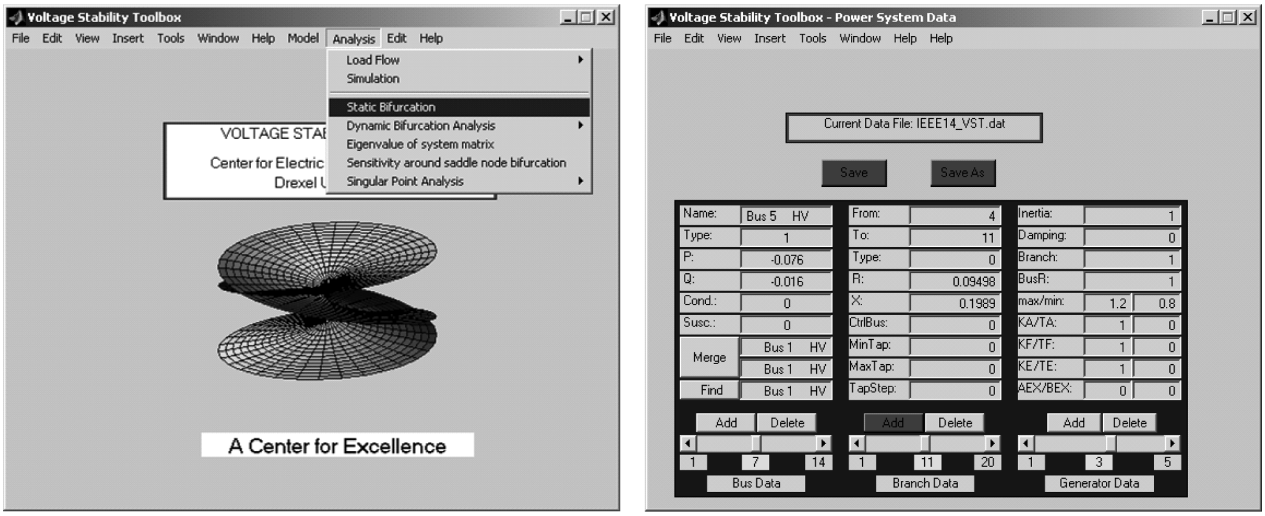
Fig. 1 illustrates the configuration of the overall software package, and Fig. 2(a) shows the main window indicating the types of analysis. Sections III-A–E describe input data processing, model building and code compiling, and application modules of VST.

A. Input Data Processing

Before any computations can be performed, a data file that describes the power system must be read, processed, and loaded to Matlab's workspace. Currently, VST supports a modified VST data format. The user provides the IEEE Common Data Format, and VST automatically converts the IEEE data format to its customized format. The data structure of VST includes required bus data, such as bus types (swing, PV, or PQ buses), real and reactive power injections into buses, transmission line data (resistance and reactance), and generator data (inertia and damping). VST also allows the user to view and modify the selected system data via an edit interface illustrated in Fig. 2(b). Bus data, branch data, and generator data can be easily modified; and new bus, or branch, or generator can be added into an existing system.

B. Model Building and Code Compiling

In VST, symbolic computation is used to build the classical power system equations and associated Jacobian expressions and to obtain computational modules for the bifurcation analysis. The model-building process implemented in VST could



a) b)

Fig. 2. (a) Main window of VST showing the analysis; b) edit window of VST.

```
n_gen:=1:
m_pv:=0:
s_pq:=1
f1:=array(1..1,1..1):
f2:=array(1..1,1..1):
j:=array(1..2,1..2):
f1[1,1]=(((((b[2,1]*e[2])*(e[1]*sin(th[1])))+((cos(th[1
])*e[1])*(e[2]*g[2,1])))+(e[2]*(e[2]*g[2,2]))) - p[2]):

j[1,1]=(((b[2,1]*(e[1]*sin(th[1])))+((cos(th[1])*e[1])*
g[2,1])))+((e[2]*g[2,2])+(e[2]*g[2,2])):

j[1,2]=(((b[2,1]*(e[2])*(e[1]*cos(th[1])))+((((-1)*sin
(th[1]))*e[1])*(e[2]*g[2,1]))):

f2[1,1]=(((((b[2,1]*e[2])*(e[1]*cos(th[1])))+((sin(th[1
])*e[1])*(e[2]*g[2,1])))+((b[2,2]*e[2])*e[2])) - q[2]):

j[2,1]=(((b[2,1]*(e[1]*cos(th[1])))+((sin(th[1])*e[1])*
g[2,1])))+((b[2,2]*e[2])+(b[2,2]*e[2]))):

j[2,2]=(((b[2,1]*e[2])*(e[1]*((-1)*sin(th[1]))))+((cos
(th[1]))*e[1])*(e[2]*g[2,1]))):
```

a)

```
t1 = data[2]*data[0];
t2 = cos(x[0]);
t4 = sin(x[0]);
t5 = t4*data[0];
t9 = data[2]*x[1];
t10 = data[0]*t2;
t11 = t9*t10;
t12 = x[1]*data[6];
t13 = t5*t12;
t14 = pow(x[1],2.0);
t17 = t9*t5;
t18 = t10*t12;
J[3] = t1*t2+t5*data[6]+2.0*data[4]*x[1];
f[1] = t11+t13+data[4]*t14-param[2];
J[1] = -t17+t18;
J[0] = t11-t13;
J[2] = t1*t4+t10*data[6]+2.0*x[1]*data[8];
f[0] = t17+t18+t14*data[8]-param[1];
```

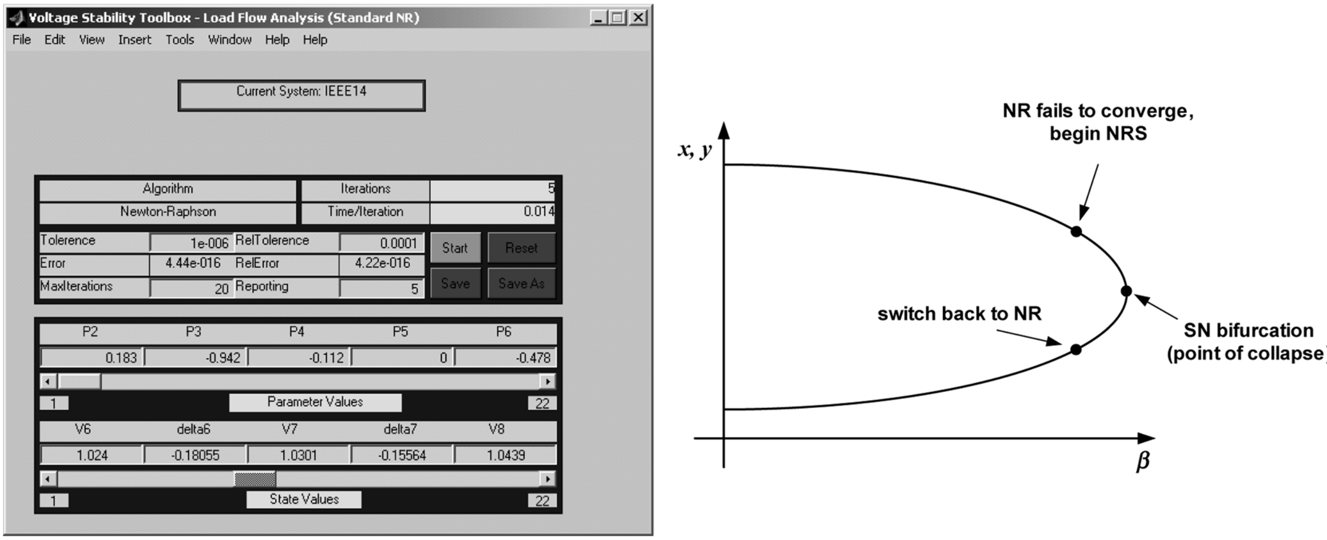
b)

Fig. 3. (a) Power system governing equations and Jacobian expressions and (b) optimized source code.

be summarized as follows. First, a power system data in IEEE Common Data Format is imported, converted to the VST data format, and edited, if necessary, in the Edit GUI shown in Fig. 2(b). Second, a core (classical) network model and associated Jacobian expressions, including the second order derivatives, are built using symbolic constructions coded in a C source file that has been compiled and is available to Matlab as a function ("MEX function"). The input to this function is the system data, and its output consists of the equations of the classical power network model and Jacobian expressions in either C-form or MAPLE-form. This routine is very efficient for model assembly. The MAPLE-form equations and Jacobian matrix for a three-bus power system are shown in Fig. 3(a). The MAPLE-form equations are further processed using the Matlab symbolic toolbox. This process can include modification of the classical model (addition of exciters, governors, tap changing transformers, etc.), and also optimization of the computational sequence by identification and elimination of repeated expression. Finally, MAPLE functions convert the optimized equations to C source code that compiles as a MEX function callable from Matlab for numerical computations. The optimized C code for the three-bus power system is given in Fig. 3(b).

C. Load-Flow Analysis

VST implements the NR algorithm to compute the load-flow solutions at a given set of bus injections. The load-flow analysis module has easy-to-use GUI as shown in Fig. 4(a). The GUI



a) b)

Fig. 4. (a) GUI for load flow analysis and (b) a three-stage process for the computation of equilibrium and bifurcation points.

for load-flow analysis allows the user to set control variables of the NR algorithm, such as maximum number of iterations, error tolerances, and to specify real/reactive power demand at the load buses (*Parameter Values*), and the initial conditions of voltage magnitudes and angles to be solved (*State Values*) through the editable fields at the bottom of the interface. The MEX function used for the NR method has the following calling syntax:

$$[\,f \quad J\,] = \text{function}(\text{data}, X, \text{param}) \tag{2}$$

where "data" is a data set consisting of the bus data, branch data, and generator data needed to run standard load-flow calculations; X is the vector of voltage magnitudes and angles; and "param" is the vector of real/reactive power demand at the buses specified by the user; f is a vector of load-flow mismatches; and J is the Jacobian matrix that includes first-order derivatives only. The load flow results are displayed in the field named *States Values* as shown in Fig. 4(a). The resulting voltage magnitudes are in pu, and angles are in radians. The user is also able to save the load-flow results in a table format to Matlab's workspace.

D. Voltage Stability Analysis

The voltage stability analysis requires the identification of all equilibrium points (nose curve) and their stability features and associated local bifurcations (SN, Hopf, and SI bifurcations) for a given load increase pattern [1]. VST implements load-flow calculations that function up to the point of collapse (SN bifurcation point). Conventional numerical methods for computing equilibria, such as the NR method, must be modified to obtain reliable results near bifurcation points. In VST, a three-stage load-flow method has been implemented to obtain entire nose curve for a given load increase pattern. First, the standard NR method is used until it fails to converge. Then, it automatically switches to the NRS method to find load-flow solutions at and around the SN point. The NRS method is a direct method that uses the second-order derivatives to compute the SN bifurcation point [33]. After passing through the bifurcation point, the standard NR method is switched back to compute low voltage solutions. Therefore, by using VST, the whole equilibria can be easily traced, and the voltage stability characteristics can be found. The three-stage tracing process is illustrated in Fig. 4(b). In the NRS method, the load-flow equations and the Jacobian matrix are evaluated numerically through the following callable MEX function in the VST.

$$[\,f \quad J\,] = \text{function}(\text{data}, X, \text{param}, v). \tag{3}$$

Compared with the MEX function of load-flow analysis given by (2), here an additional input v is included. This new input vector is the null space spanning vector (right eigenvector) of Jacobian matrix at the SN bifurcation point. The matrix $J = [\,J_1 \quad J_2\,]$ is the Jacobian matrix containing first (J_1) and second-order derivatives (J_2).

Fig. 5(a) shows the GUI of dynamic bifurcation analysis that implements the three-stage algorithm. This GUI enables users to set absolute and relative tolerances, maximum number of iterations, and initial values for both power injections and state variables. When it is run, it gives the set of equilibrium points (nose curves) with respect to parameter changes. In VST, the concerned parameters are the real/reactive loads at the buses that can vary according to the following formula:

$$\begin{aligned} P &= P_0 + \text{alpha}^{*}\text{direction}P \\ Q &= Q_0 + \text{alpha}^{*}\text{direction}Q \end{aligned} \tag{4}$$

where P_0, Q_0 and P, Q are vectors of initial and actual real and reactive power, respectively; "direction$P$" and "direction$Q$" are vectors of searching directions, which can be set to be positive, zero, or negative for each bus. Therefore, one may conveniently increase load or increase generation at a bus and/or at a group

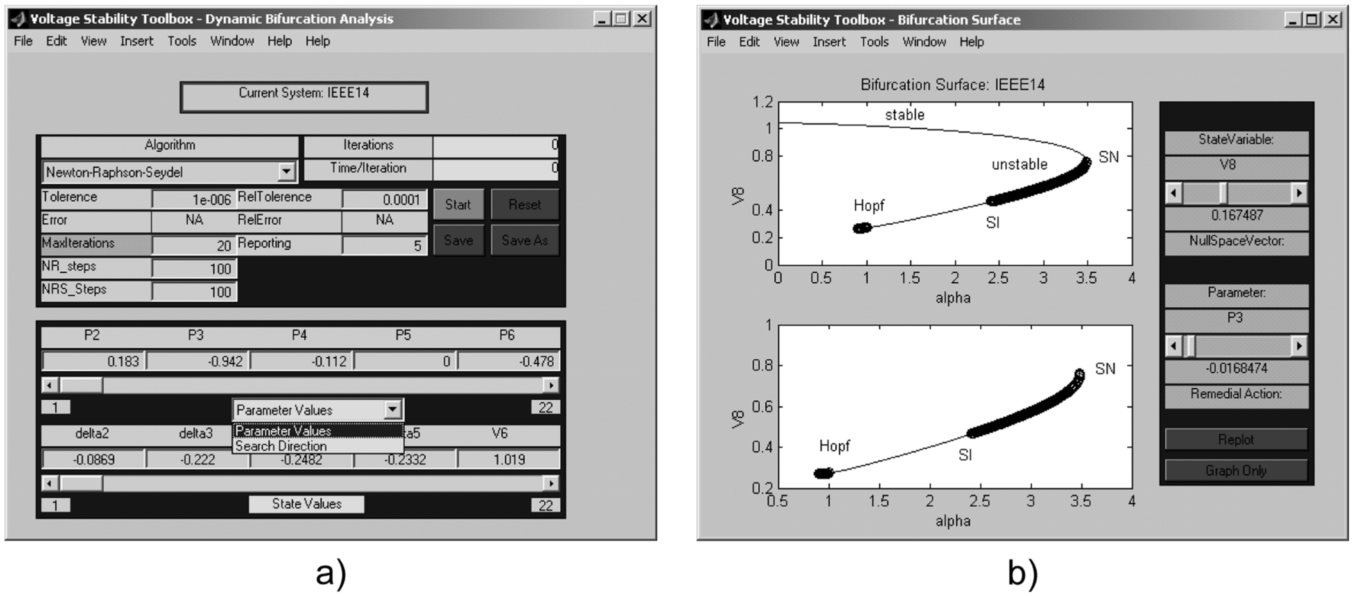


Fig. 5. (a) GUI for dynamic bifurcation analysis and (b) bifurcations for the IEEE 14-bus system.

of buses, while the power factor is kept constant or varying. At present, a fixed-step size is used at the NR stage, meaning that "alpha" can be increased or decreased by a fixed-step size. However, at the NRS stage, the step size is changed automatically by VST according to the second-order derivatives.

The dynamic bifurcation analysis not only computes equilibrium points but also identifies their small-signal stability characteristics by computing and checking the eigenvalues of the reduced system Jacobian matrix at each equilibrium point along the nose curve. Unlike other software packages, such as PST and Simulink-based tools, eigenvalues are computed using the symbolically computed Jacobian matrices, thus ensuring high-precision results. The reduced-system Jacobian matrix $[A_{\rm sys}]$ is defined as follows:

$$[A_{\rm sys}] = [D_x f] - [D_y f]\,[D_y g]^{-1}\,[D_x g]. \qquad (5)$$

The dynamic bifurcations (Hopf and SI) are determined by monitoring the eigenvalues of the system matrix as the system moves from one operating point to another along the nose curve with changes in the parameter α. On the other hand, the static bifurcation analysis implements only a two-stage (NR$\rightarrow$NRS) algorithm to compute equilibrium points up to the tip of the nose curve (the SN bifurcation point) and does not determine the small-signal stability of the operating points. Therefore, it is called static rather than dynamic bifurcation analysis.

Fig. 5(b) illustrates the output window of the dynamic bifurcation analysis. For the simulation, the real power demand at bus 8 of the IEEE 14-bus system is increased according to (4). The voltage magnitude at bus 8 is selected to show how the system equilibria and stability properties change with parameter variations. [Note: any of the voltage magnitudes or phase angles can be plotted by using the slider in the right part of the window.] Observe that along the nose curve, SN, Hopf, and SI bifurcations are observed. The SN bifurcation is the tip of the nose curve at which parameter $\alpha = 3.487$. Beyond this parameter value, no solution is available to the load-flow equations. In the lower part of the nose curve, the SI and Hopf bifurcations are observed. The SI bifurcation happens at $\alpha = 2.409$, while two Hopf bifurcations occur at $\alpha = 1.015$ and $\alpha = 0.904$.

In VST, small-signal stability analysis of operating points could be easily performed since the eigenvalues of the system matrix, evaluated at any operating point along the nose curve, are available from the dynamic bifurcation analysis and stored in the Matlab's workspace. Fig. 6 shows the loci of the eigenvalues of the system matrix for the same load increase scenario indicating the occurrence of Hopf bifurcation.

E. Dynamic Simulations

The TD simulation program is designed to analyze the system's local dynamic behavior around an operating point. The TD simulation module enables users to verify the small-signal stability features of any operating point. For the DAE model of power systems, the load-flow equations for PV and PQ buses have to be solved in advance of solving the differential equations of the generators. The TD simulation program implements an ordinary differential equation (ODE) solver combined with an algebraic solver at each iteration step. A fourth-order Runga–Kutta method is used as an ODE solver, while an NR procedure is implemented as an algebraic solver to update the load bus voltage magnitudes and angles at each integration step.

The GUI for TD simulation is shown in Fig. 7(a). From the GUI, any operating point along the nose curve can be selected by using the slider at the top of the window (*CurrentPointNumber*) as the initial operating point for simulation, and the excitation (disturbance) can be set by using the editable fields (*StateValues*). The simulation results for the desired variable can be selected by using the slider (*SelectedVariable*) and plotted in another window [Fig. 7(b)]. Fig. 7(b) shows simulation results initiated at a stable operating point ($\alpha = 2$) for the generator 2 angle.

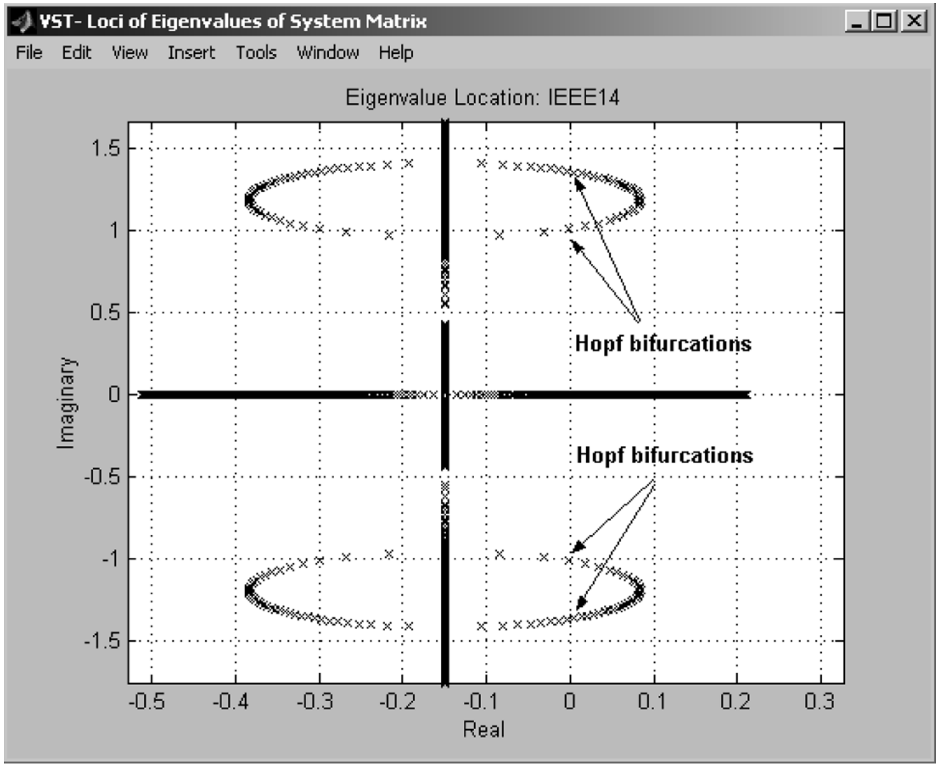


Fig. 6. Eigenvalues of the reduced system matrix.

IV. Educational Use of VST

This section describes how VST was used in power engineering courses in the Electrical and Electronics Engineering Department of Nigde University. These courses are Power System Analysis II and Power System Stability and Control.

In the undergraduate course Power System Analysis II, VST was mainly used to enhance the teaching load flow analysis and related computational issues. After the standard material on load flow modeling (bus model, admittance matrix, and power mismatches) and solution algorithms (Gauss–Seidel, NR, and decoupled algorithms) are covered in the class, the instructor uses VST for in-class demonstration for various sized prebuilt and compiled power systems ranging from a two-bus system to the IEEE 118-bus system. After the demonstration, students use VST for the the IEEE 14-bus system to investigate the effects of load and/or generation increase/decrease on the voltage profile and to determine the most sensitive bus with respect to a load increase. Students through this exercise should have a basic understanding of the network models, fundamental bus models, and how the voltage profile of the system is affected by the variation in load/generation. At the same time, they should have gained some understanding of the NR algorithm and the concept of convergence. During the class, students are also asked to find out how the solution and convergence might change with some of the NR algorithmic parameters, such as maximum number of iterations, error tolerances, and initialization. After having enough experiences with VST, some of the following exercises are assigned to students for the IEEE 14-bus system:

- increase the active and reactive load demand at any of PQ buses while keeping the power factor constant, determine bus voltage magnitudes and angles, and plot results using bar-chart-plotting option of Matlab;
- write a Matlab code to determine the active and reactive power flows at each line;
- investigate the effect of the line loss between buses 4 and 12 on the voltage profile of the system;
- find loading conditions to experience convergence problems in the NR algorithm.

Fig. 8 shows the voltage profile (magnitude and phase angle) of the IEEE 14-bus system when the load demand at bus 9 was increased to twice of the original demand. These plots are the result of the first assignment given for which students need to explore the LF program in VST to extract correct bus data (note that PV bus magnitudes are constant and are not displayed in the LF's GUI shown in Fig. 4(a); this data must be extracted from the bus data file) and to find the plotting options of Matlab. These plots help students understand that load increase in a particular bus (bus 9 in this case) not only affects the voltage profile of this bus, but also influences adjacent buses (buses 6, 8, 10, and 14) connected to the bus 9. Moreover, examining the load-flow solutions for various loading conditions (i.e., increase in real power only, or reactive power only, or both) students clearly see that variation in reactive power load mostly affects the voltage magnitudes, while variation in real power load influences the phase angles (P-δ and Q-V interactions).

Power System Stability and Control, a graduate course, mainly focuses on bifurcations and voltage stability, small-signal stability, transient stability, and control of active and reactive power. For this course, the bifurcation analysis and TD simulation applications of VST are used for class demonstration to illustrate to students the concept of multiple

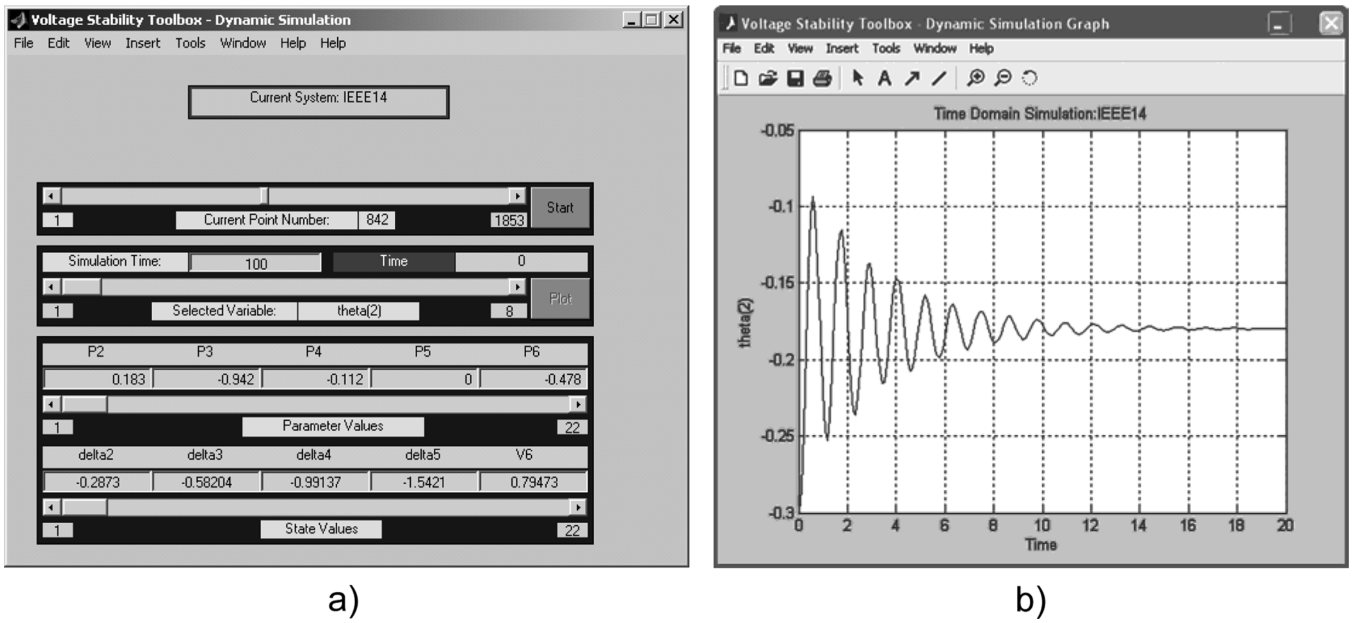


Fig. 7. (a) GUI for dynamic simulations and (b) simulation result for generator 2.

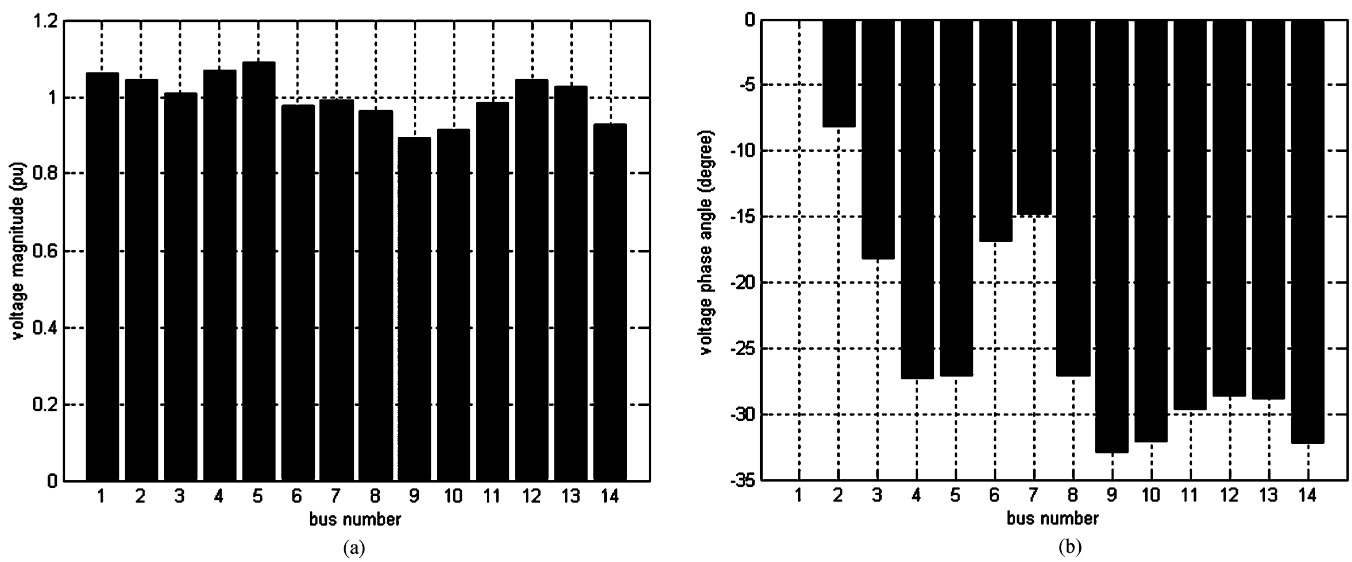


Fig. 8. Voltage profile of IEEE 14-bus system: (a) Voltage magnitude and (b) phase angle.

equilibria, loss of equilibria, small-signal stability features of the equilibria, and local bifurcations. The PV curves for a load increase pattern as illustrated in Fig. 5(b) combines the above concepts into a single graph, helping students easily grasp the idea. During the class, students are asked to choose a load increase scenario for the IEEE 118-bus system to observe the change of equilibria and stability with respect to the load increase to determine local bifurcations. They are also asked to run the TD simulation around stable and unstable operating points along the PV curve to investigate the dynamic response of the system. The graduate students are given several assignments for which the capabilities of VST could be used. The typical assignments are as follows:

- find a load/generation increase pattern so that the entire lower voltage part of the PV curve is unstable (i.e., only the SN bifurcation is observed);
- using the right and left eigenvectors information at the SN bifurcation point, determine the buses that are most sensitive to loading and find remedial action information useful for corrective control [29];
- analyze the participation factors to determine which components (state variables) are most involved in the various system modes (eigenvalues) [34];
- initialize TD simulations around the Hopf bifurcation point shown in Fig. 5(b) and determine the frequency of the oscillations.

Examples of simulations obtained by students for given assignments are presented in Figs. 9 and 10. Fig. 9(a) shows a PV curve (nose curve) whose lower part is unstable, while Fig. 9(b) depicts TD simulation for generator 5 rotor angle indicating undamped oscillations because of a Hopf bifurcation. By obtaining such simulation results students can gain hands-on

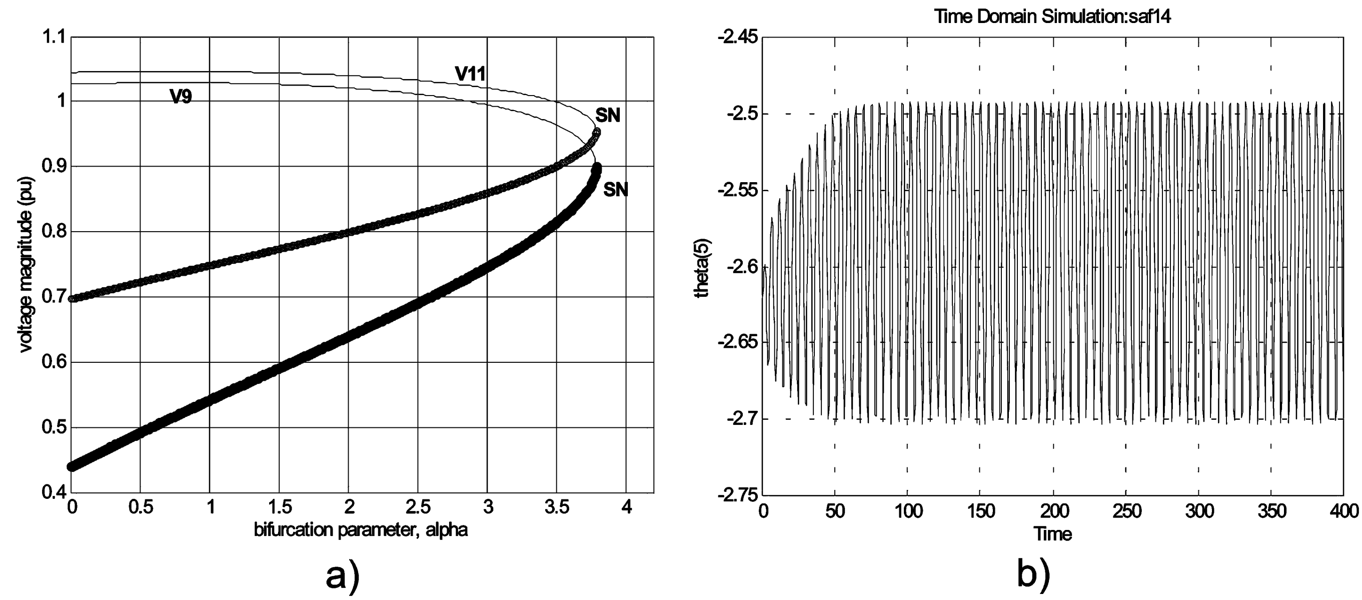


Fig. 9. Results of students' assignments: (a) PV curves whose lower part is unstable and (b) Hopf bifurcation.

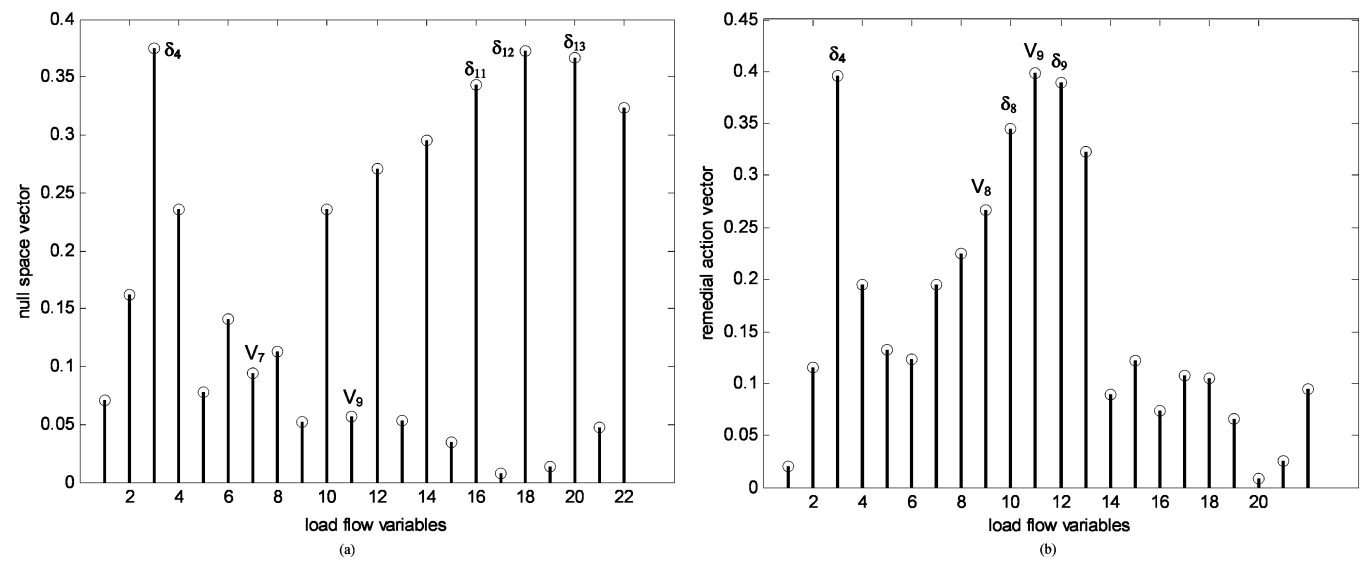


Fig. 10. Sensitivity and remedial action information: (a) Right eigenvector and (b) left eigenvector.

experience on power system dynamic behavior. Fig. 10 depicts absolute values of the component of the right eigenvector (null-space vector) and left eigenvector (remedial action vector) corresponding to the zero eigenvalue of load-flow Jacobian matrix at the SN bifurcation point. With the help of the right eigenvector information, students can easily determine which load-flow variables are most sensitive to voltage stability at the concerned bifurcation point according to the magnitudes of the null-space vector. Some of the most sensitive variables are labeled in Fig. 10. From Fig. 10(b), students can easily find remedial action information, which is useful for corrective control.

The use of VST was assessed both formally with student evaluations and informally from discussions with students. Since VST were introduced to all students within a course, no good control group is available to make a meaningful statistical assessment. The student response to the use of VST has been very positive. Students increase their understanding of load flow, voltage stability, and complicated nonlinear dynamic behavior of power systems beyond the understanding they gain from classroom lectures and textbooks. The majority of the undergraduate students indicate that having a tool that is easy to use allows them to comprehend load-flow analysis and related computational issues. Graduate students indicated that through the in-class demonstrations and assignments they had a better understanding of voltage stability and power system operation. They feel that simulated examples help them understand complex bifurcation theory and its applications into power system stability analysis. Moreover, they appreciate the open architecture of VST which allows them to modify source code for their research. The student interest is partly a result of their becoming familiar with the widely used numerical simulation environment of Matlab, which they will be able to use subsequently for their senior design projects or research.

V. Conclusion and Future Work

A voltage stability simulation tool in Matlab is presented in this paper. The developed VST is the outcome of the ongoing research conducted at Drexel University on bifurcation theory and its implementation into voltage stability analysis. VST integrates numeric and symbolic computations with powerful graphical interfaces for load flow, small-signal and transient stability, and bifurcation analysis. Simulation results show that it is a powerful and promising tool for voltage stability studies, and very helpful to understand voltage stability phenomena. VST is used for illustration purposes during the lectures and by students preparing personal assignments. The student response indicates that it can be effectively used to support power engineering education, graduate courses in particular.

Based on its modular programming structure, other element models, such as nonlinear dynamic load models and more sophisticated generator models, can be easily included in the toolbox, which will definitely improve its usage. Future work will concentrate on such issues.

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